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# **ECE 4514**

## **Digital Design II**

### **Spring 2008**

## **Lecture 18:**

# **Optimizing Area**

*A Tools/Methods Lecture*

Patrick Schaumont

# Optimization and Backend Verification

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- Previous 4 lectures (Lecture 13-16):
  - How to get Verilog mapped into hardware
  
- Next 2 lectures:
  - How to *optimize* Verilog for hardware implementation
  - Today (Lecture 17): Optimize for (smaller) area
  - Thursday (Lecture 18): Optimize for (higher) performance

# Why do we need optimization?

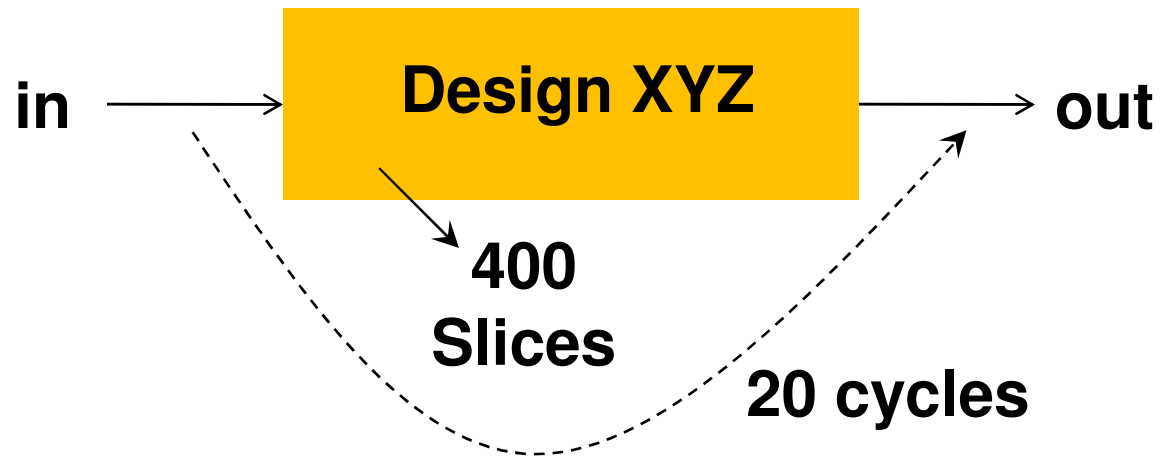
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- ❑ A given algorithm can be implemented in many different ways in digital hardware
- ❑ Each implementation is characterized by
  - A certain area (In FPGA: Slices)
  - A certain cycle budget (the amount of cycles needed to complete one iteration through the algorithm)
- ❑ The application domain and other external requirements provide *constraints* for the area or the performance (=cycle\_budget<sup>-1</sup>)
  - 'Create a circuit not larger than can fit in a Spartan3S100 FPGA ...' (area)
  - 'Create an implementation that can complete at least 500 million additions per second ..' (performance)

# Area-Delay Product: Generic Optimality Criterion

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- Area-Delay Product = Area of a circuit times the cycle budget for that circuit

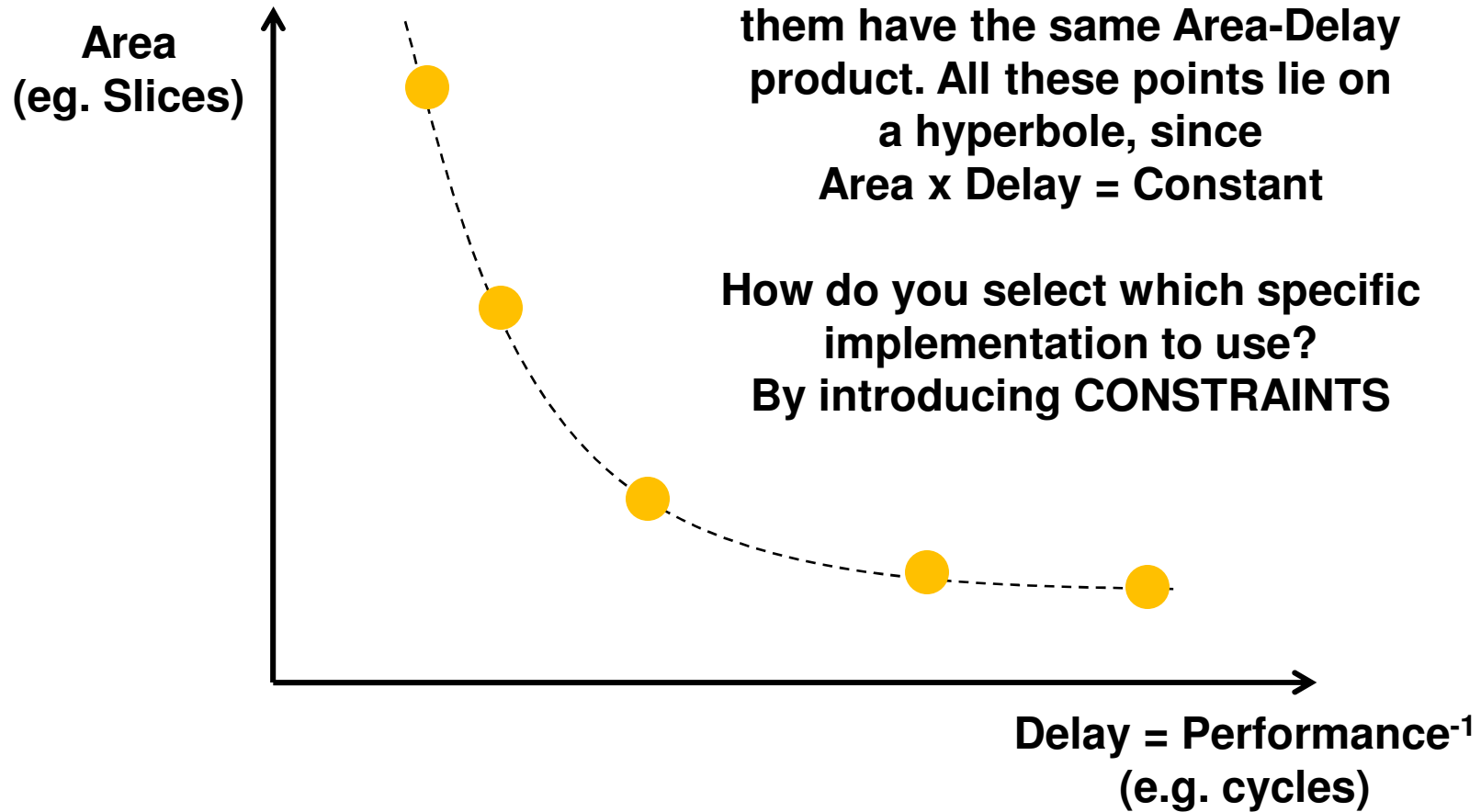


$$\text{Area-Delay} = 8000 \text{ Slices.cycles}$$

***A lower Area-Delay product is better***

# Area-Delay Product

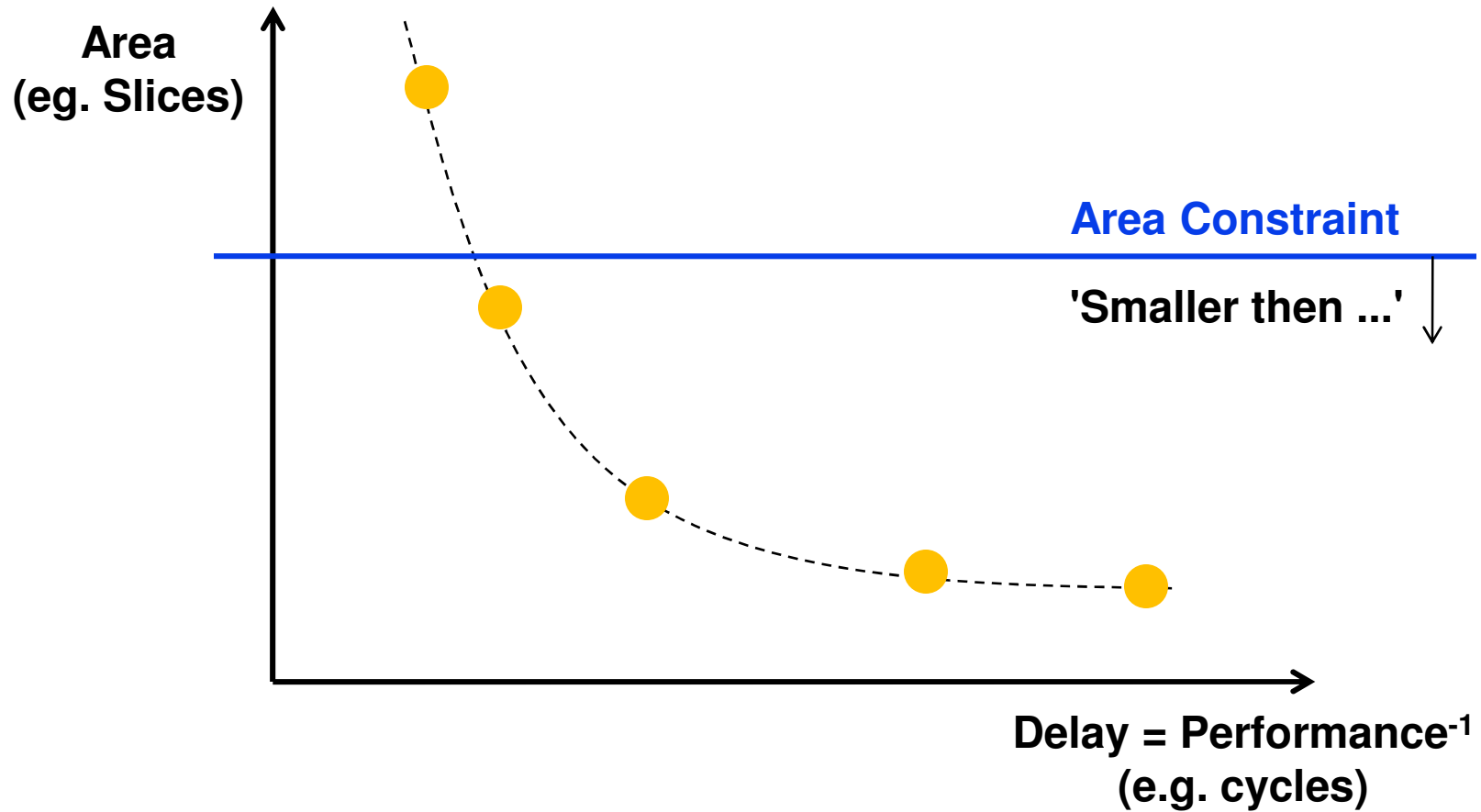
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# Area-Delay Product

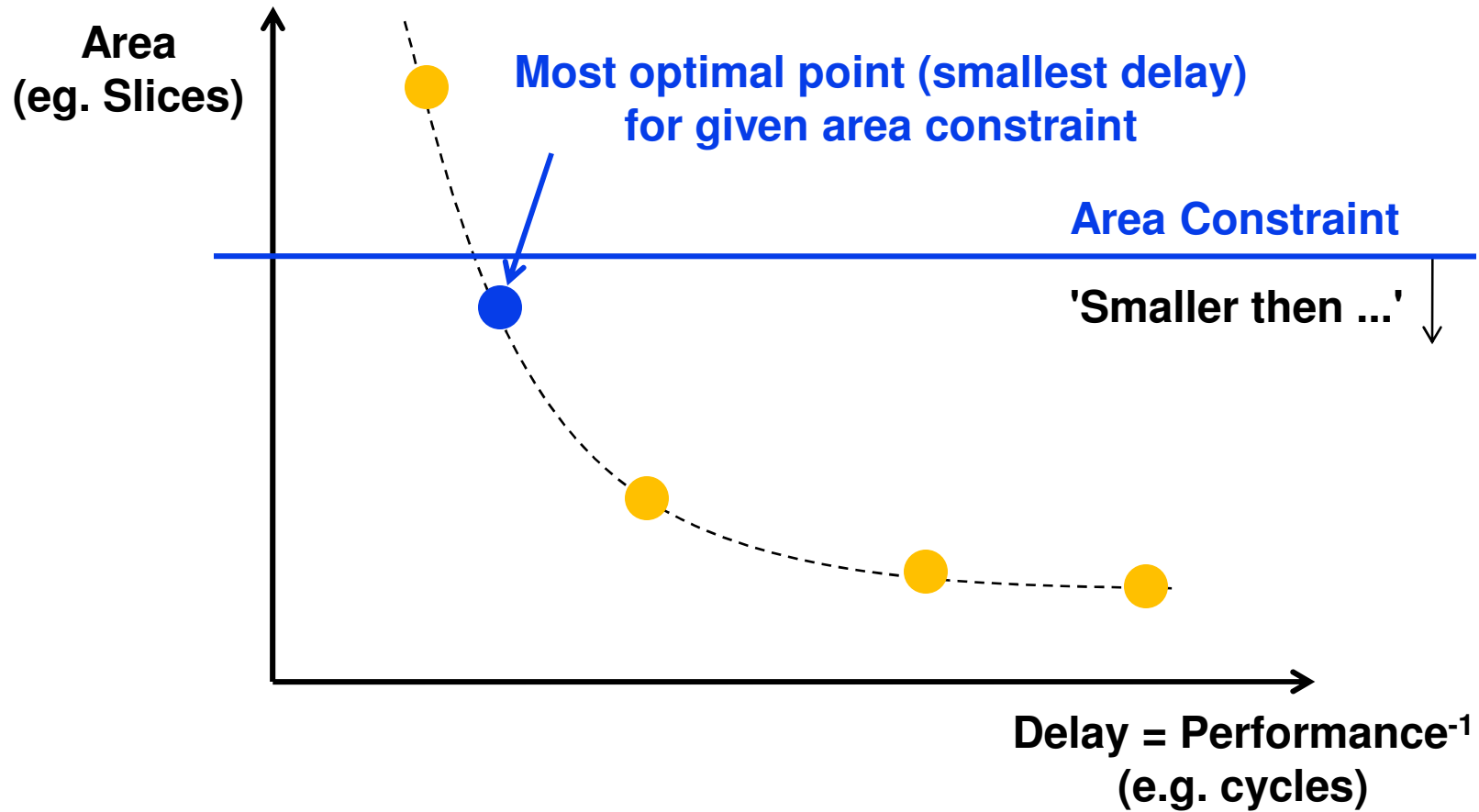
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- An Area constraint



# Area-Delay Product

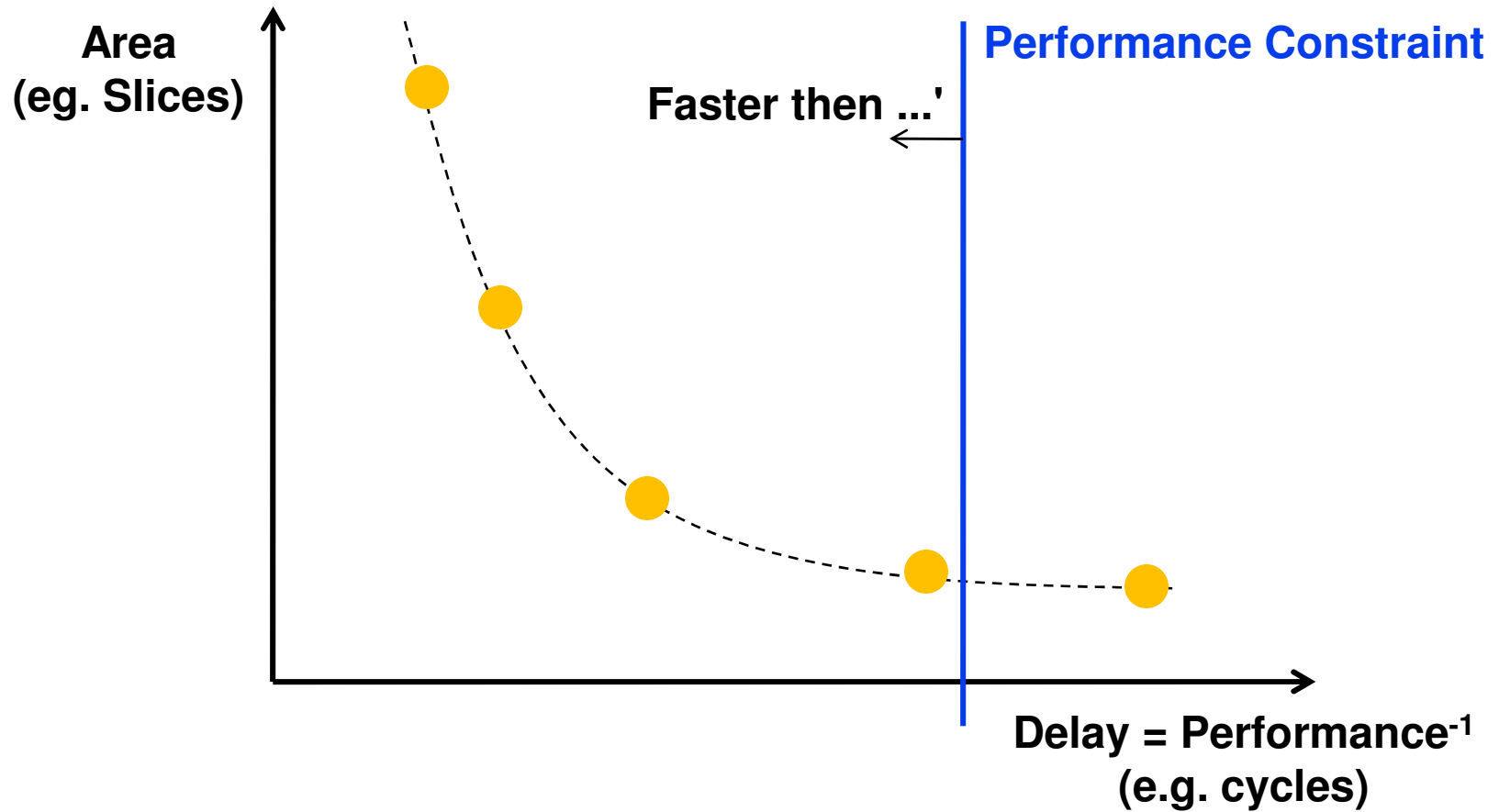
- An Area constraint



# Area-Delay Product

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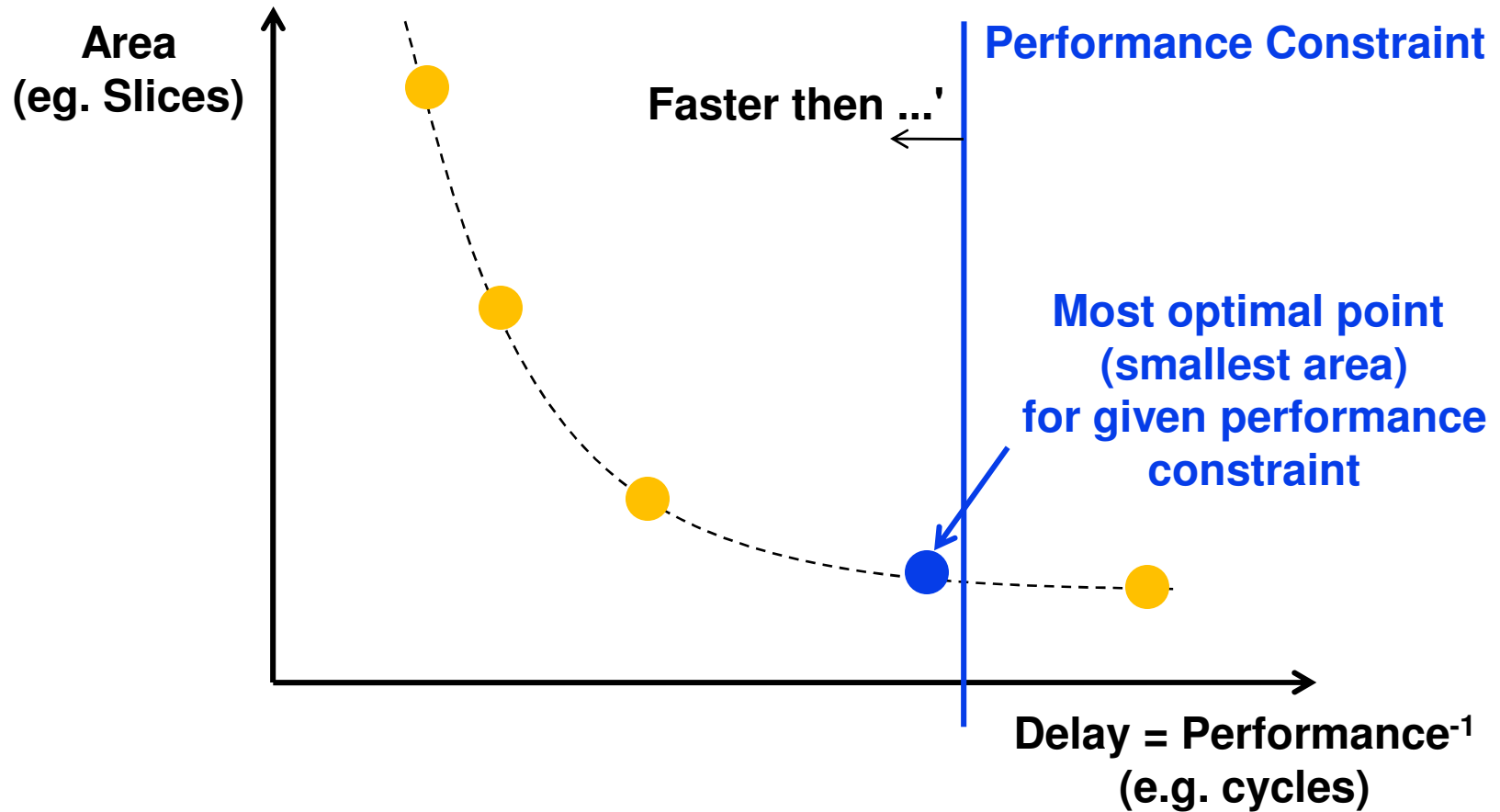
- A Performance Constraint





# Area-Delay Product

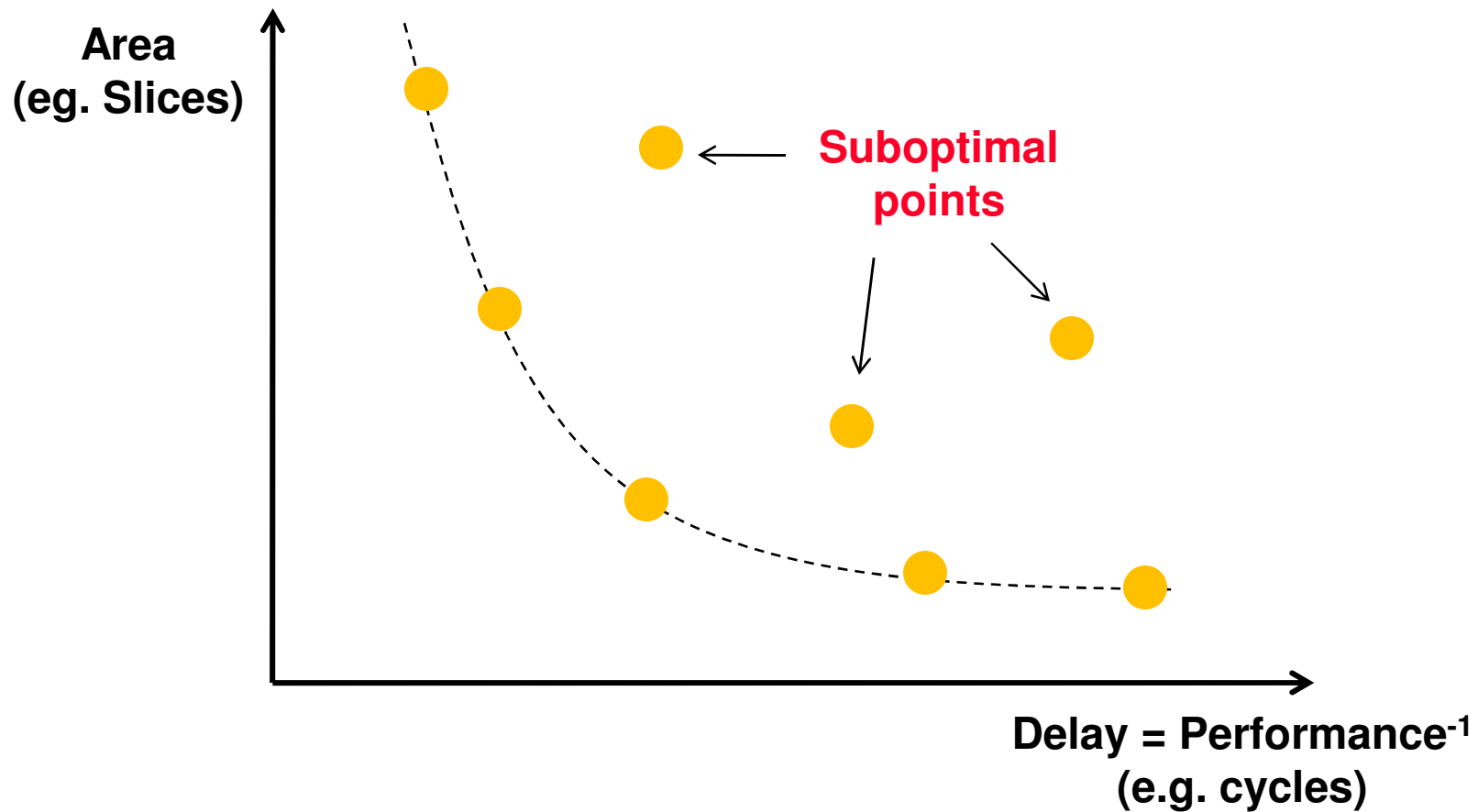
## □ A Performance Constraint



# Area-Delay Product

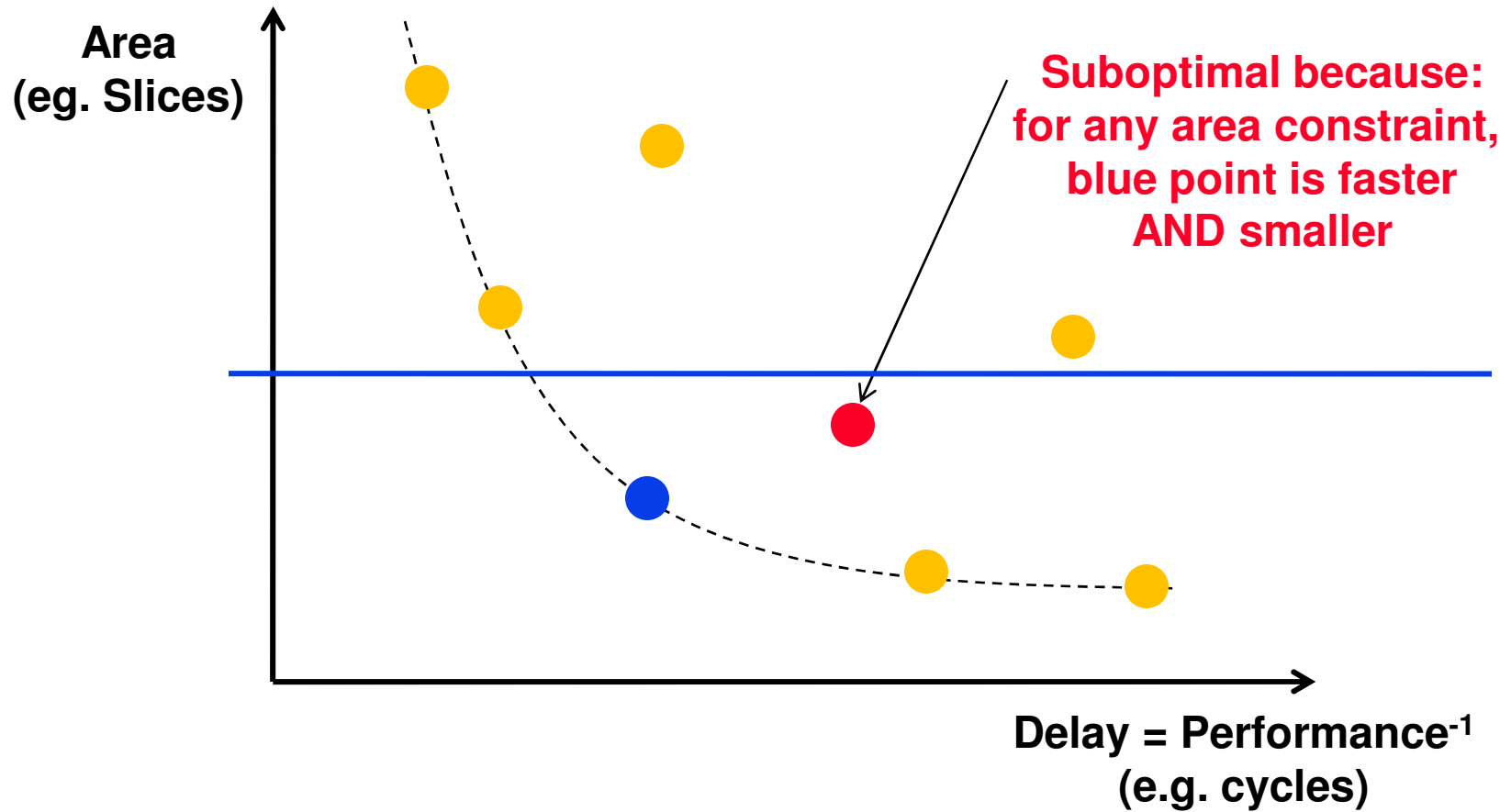
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- If no constraint is given, all points with the same area-delay product are equally optimal



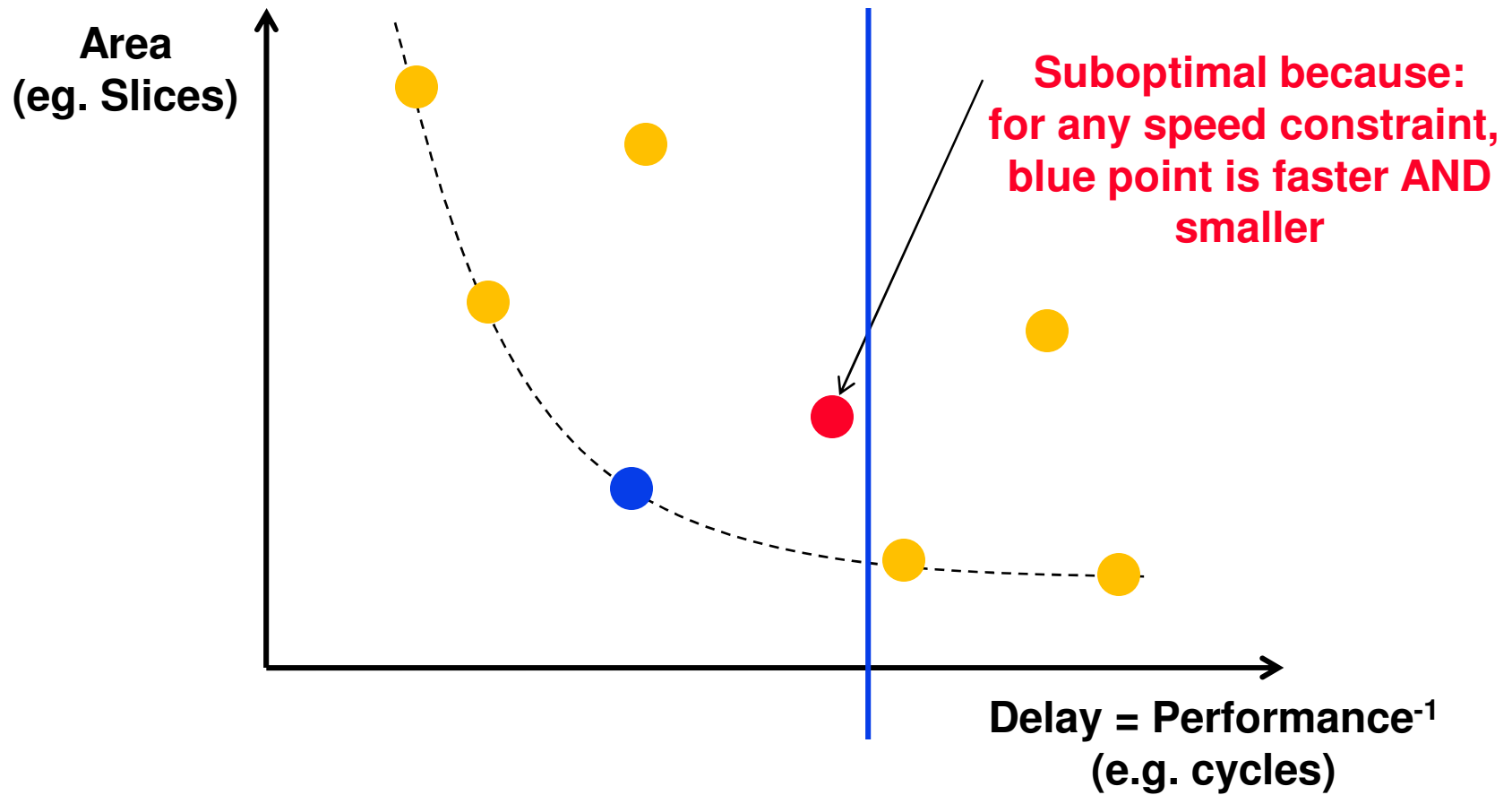
# Area-Delay Product

- Sub-optimal points



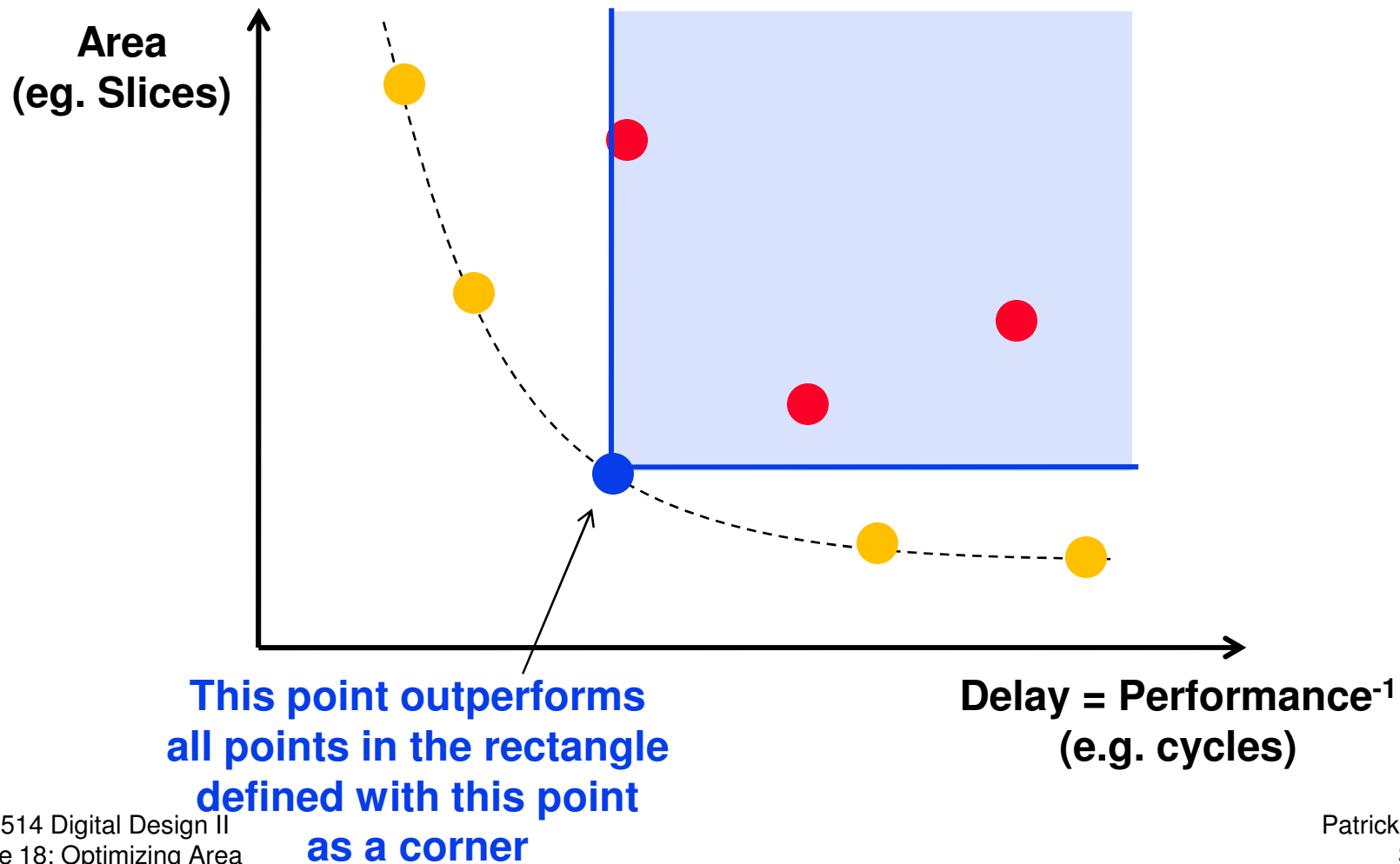
# Area-Delay Product

- Sub-optimal points



# Area-Delay Product

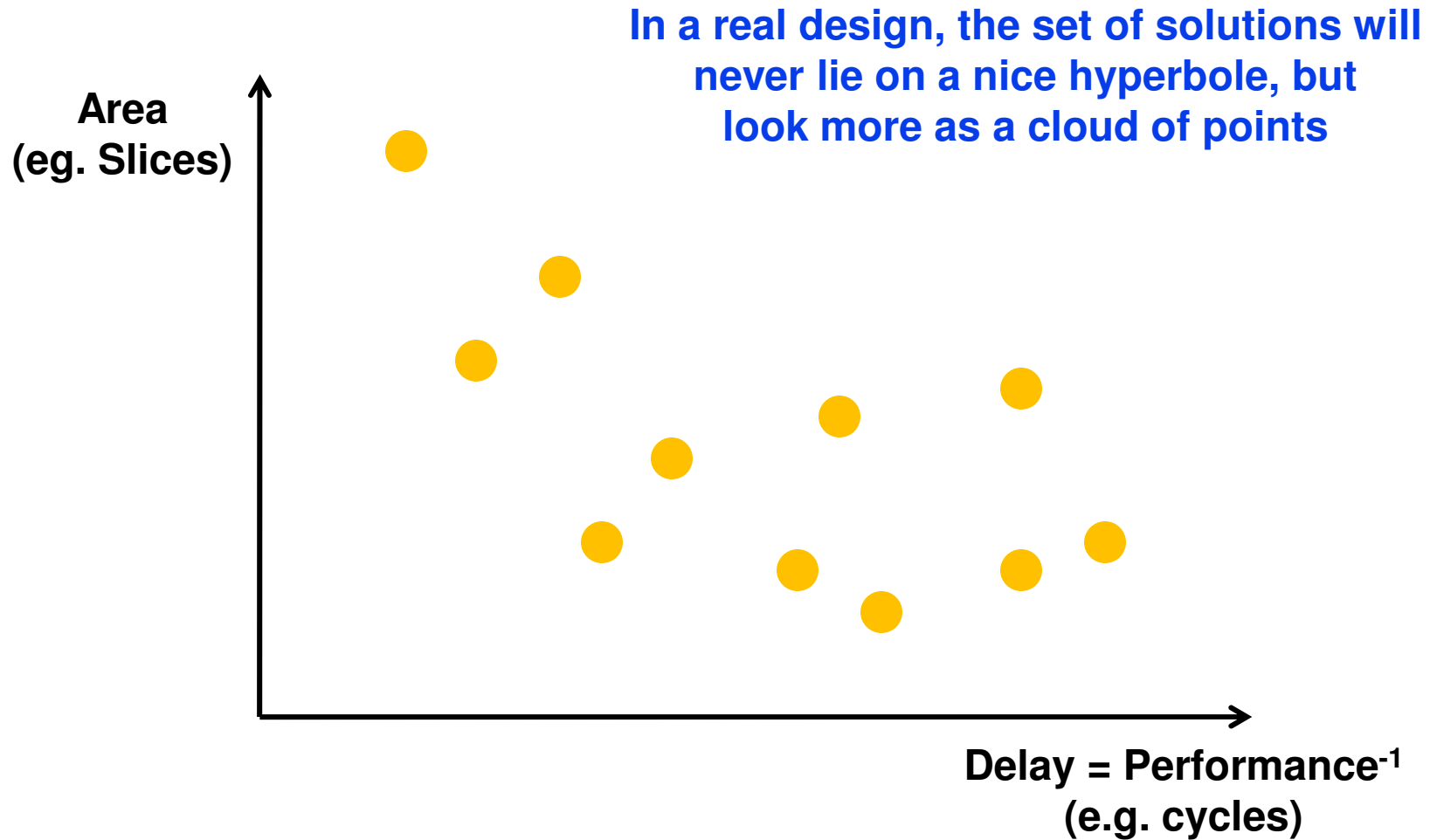
- Optimal points dominate sub-optimal points



# Area-Delay Product

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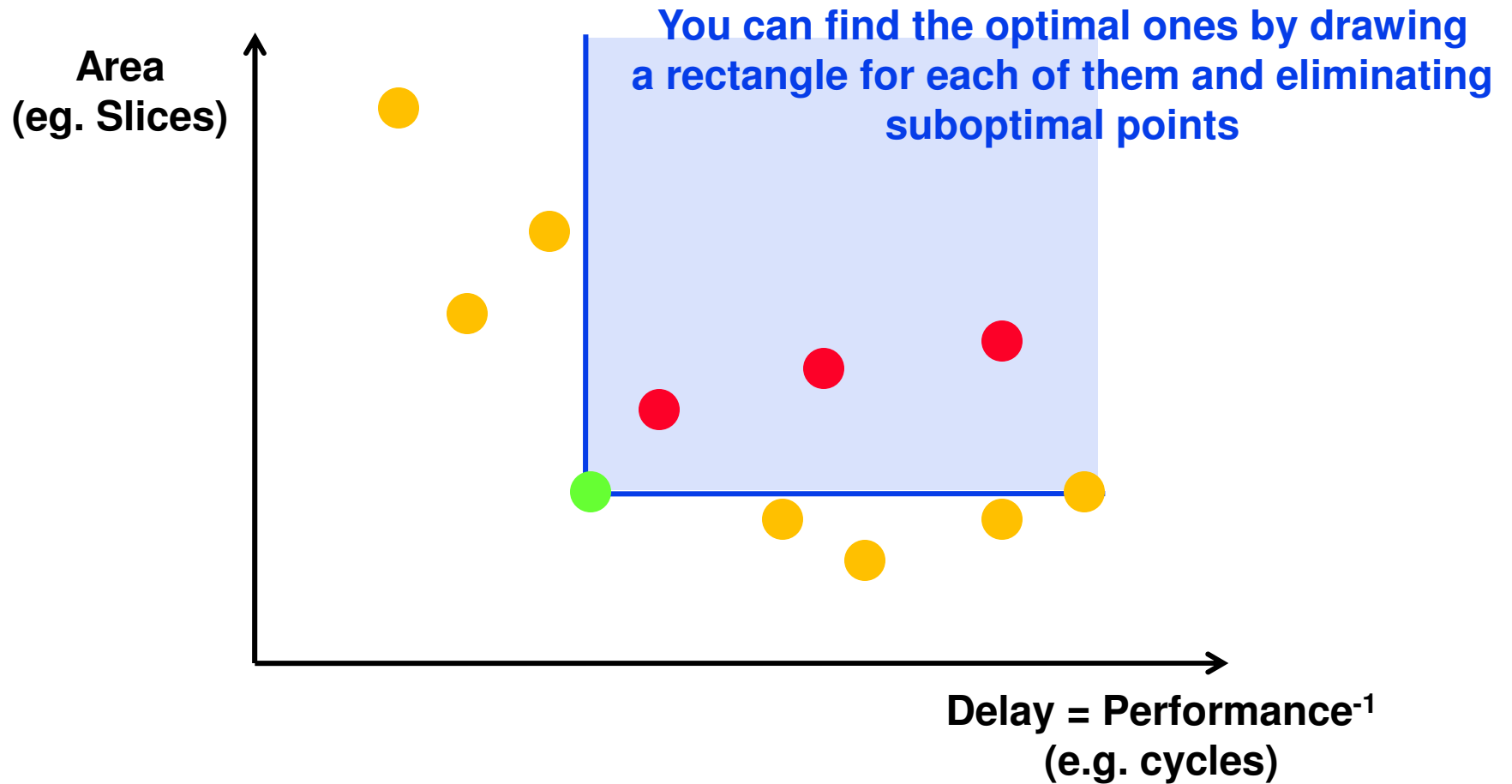
- Finding all the optimal points for a random collection



# Area-Delay Product

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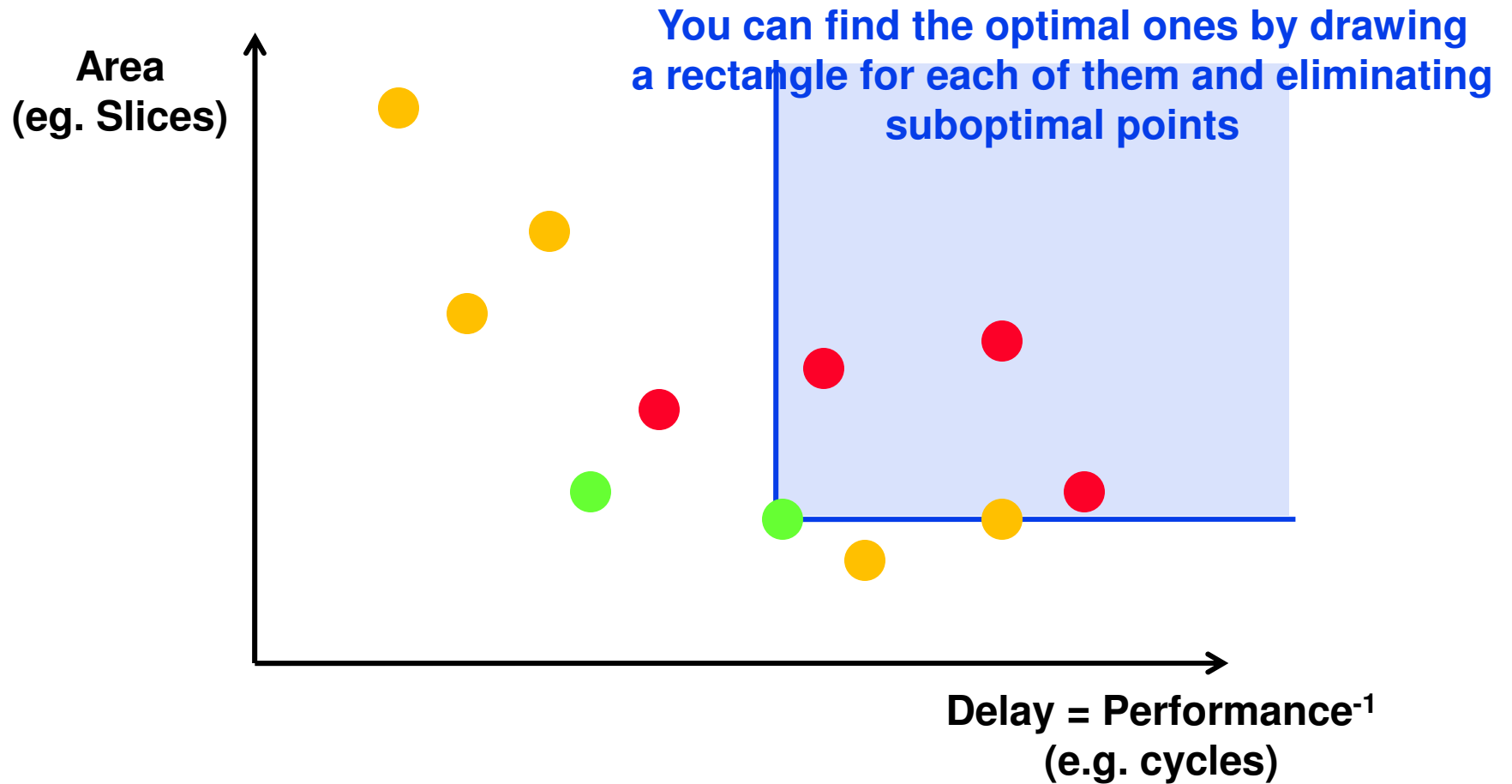
- Finding all the optimal points for a random collection



# Area-Delay Product

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- Finding all the optimal points for a random collection

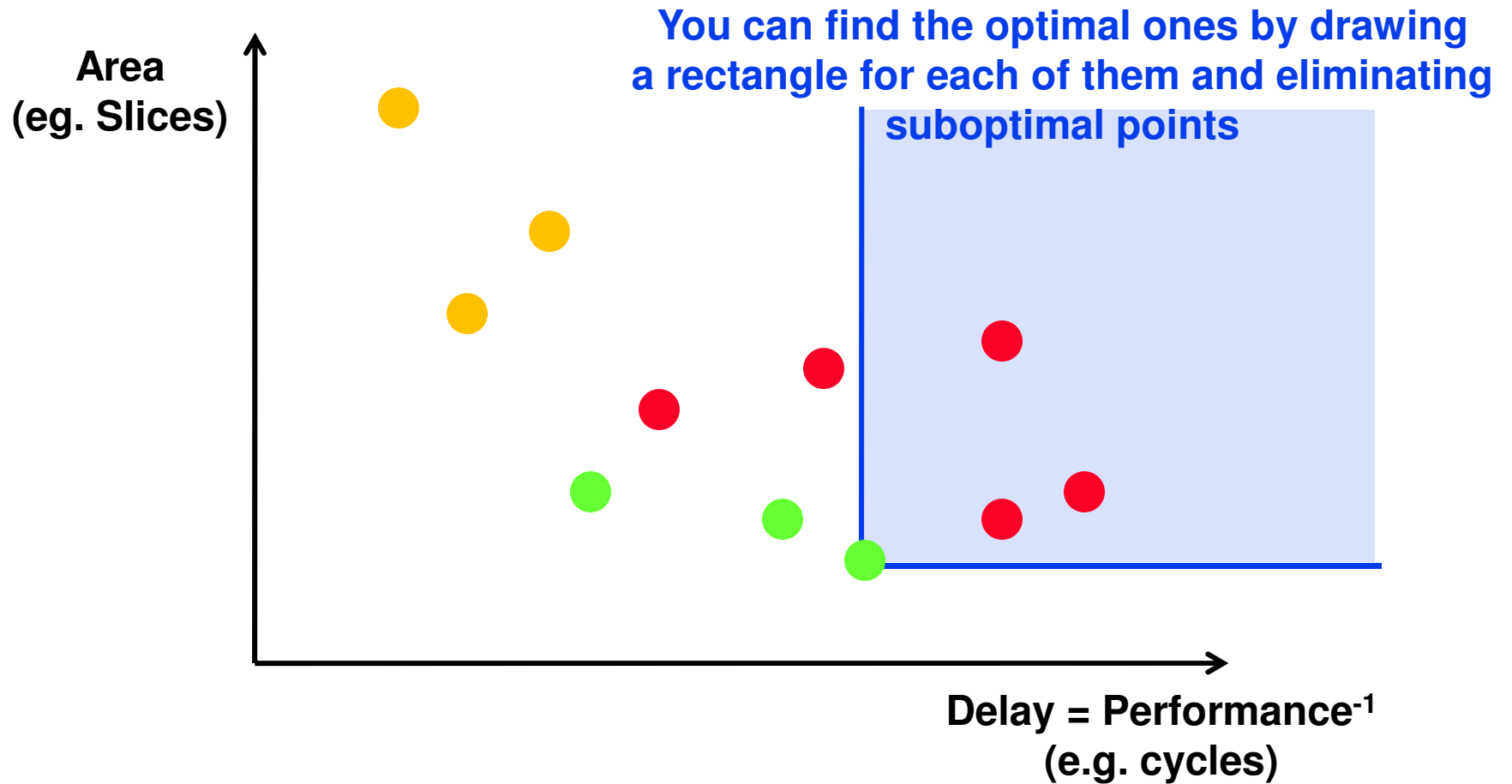




# Area-Delay Product

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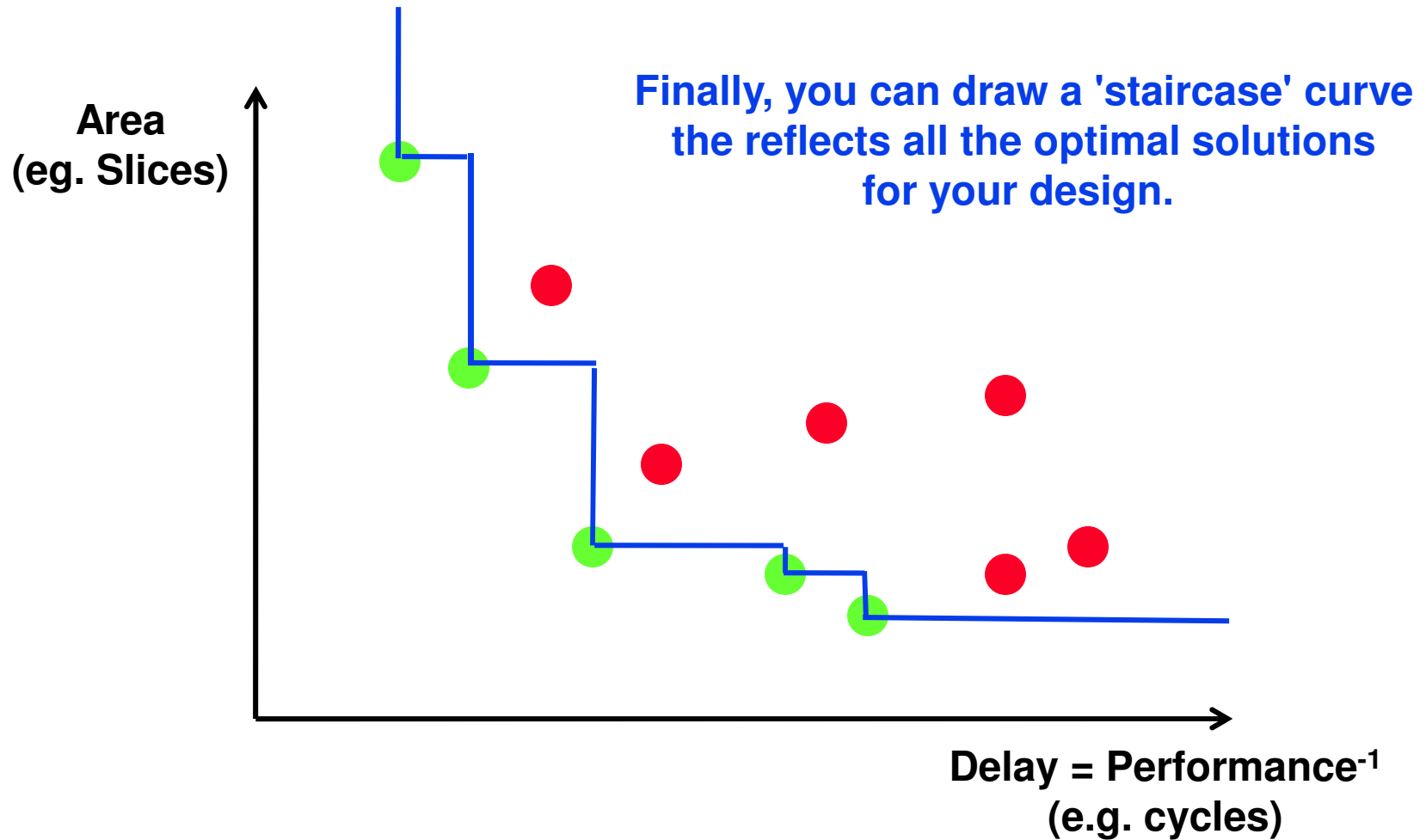
- Finding all the optimal points for a random collection



# Area-Delay Product

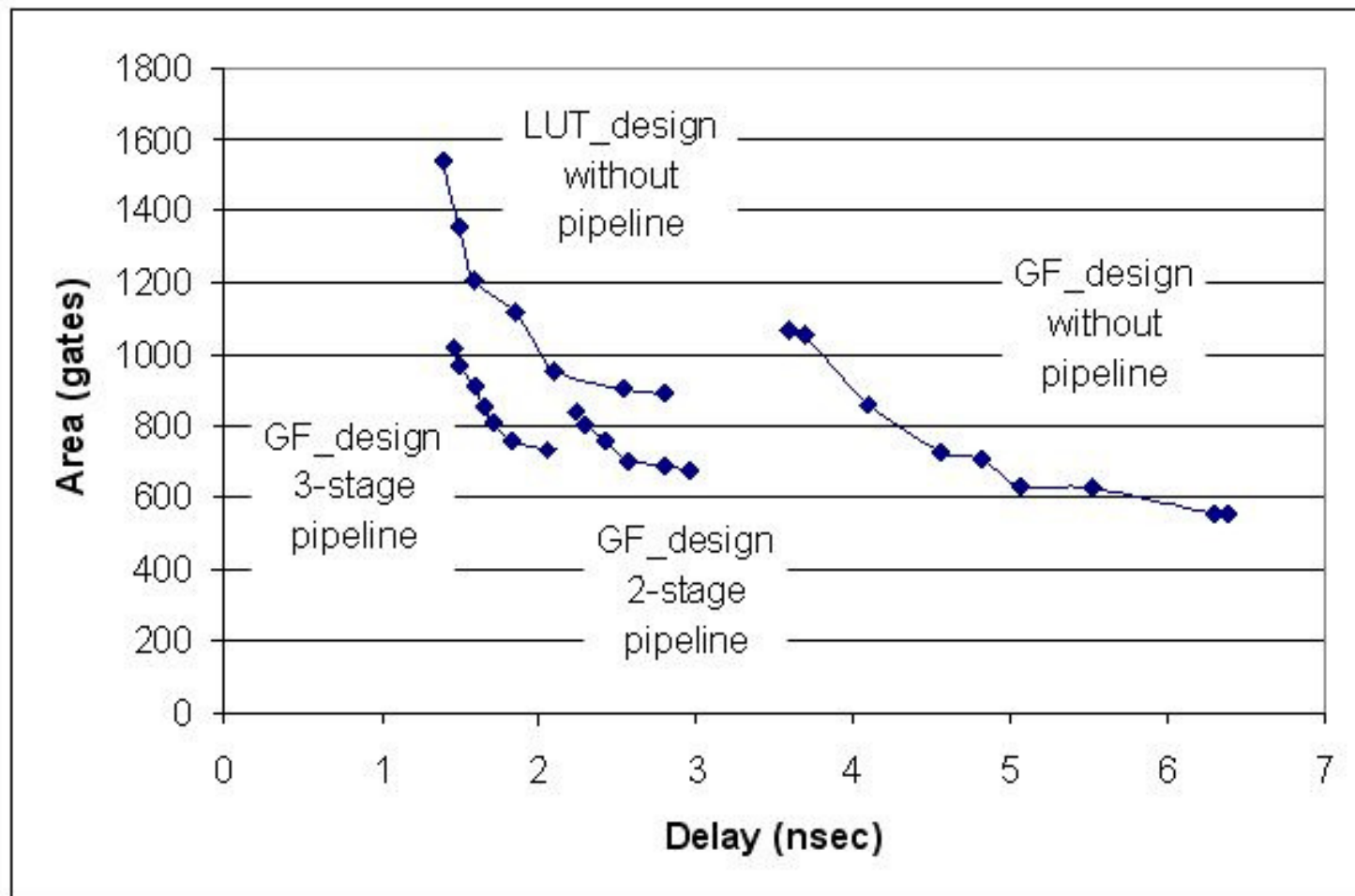
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- Finding all the optimal points for a random collection



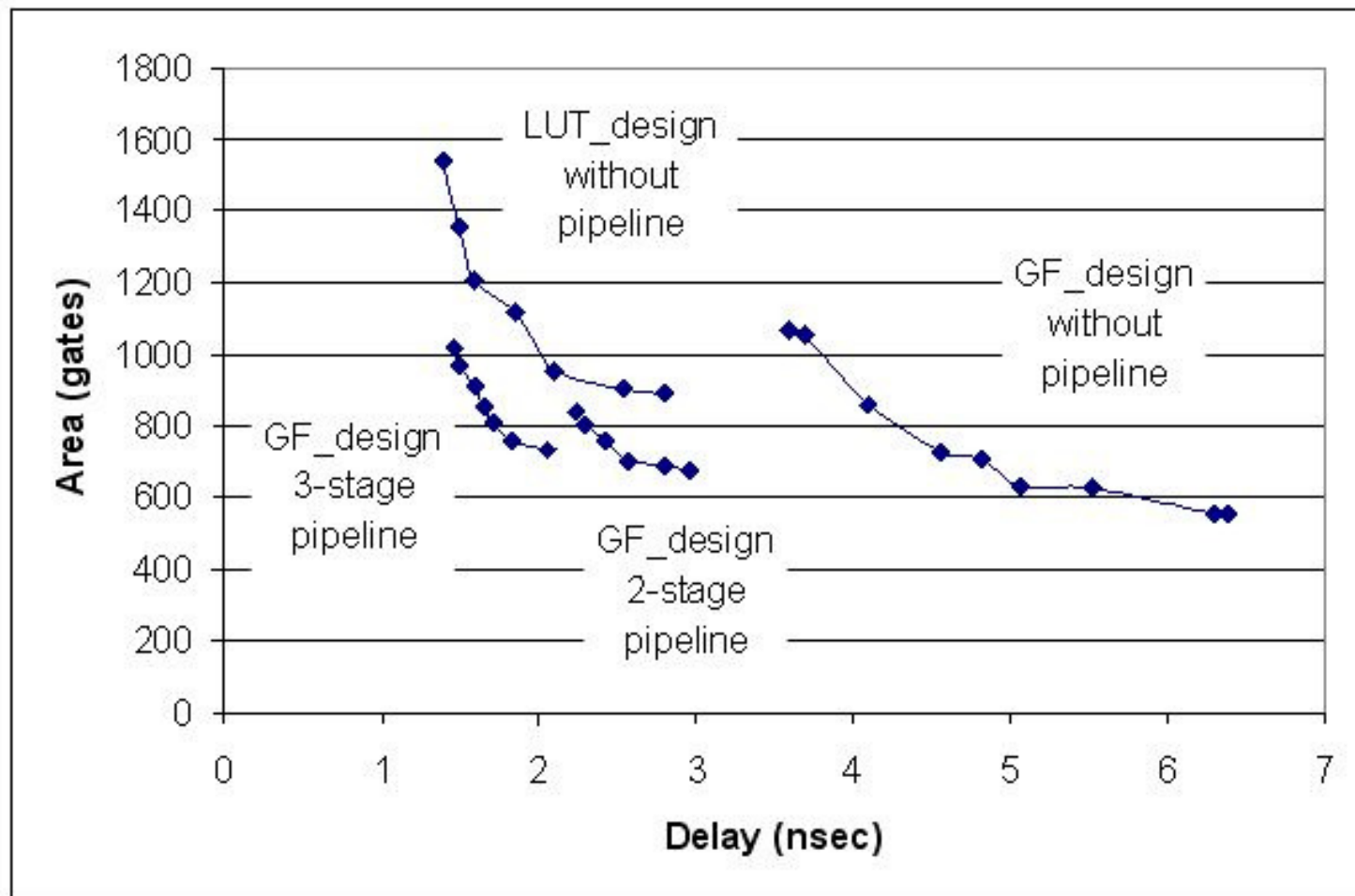
# Area-Delay Example

- Area-delay product for an AES Sbox (component of an encryption algorithm) using different architectures



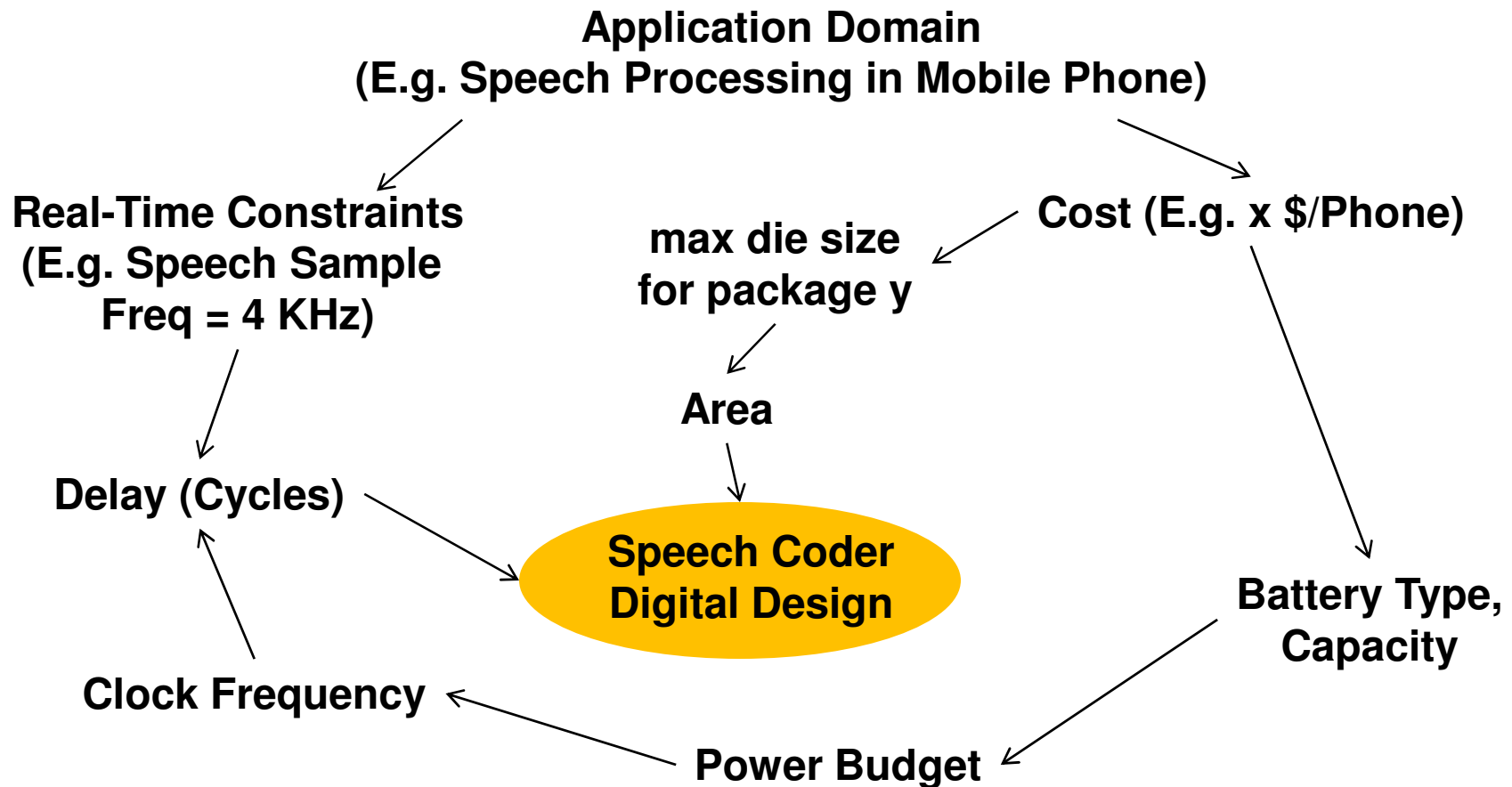
# Area-Delay Example

- Area-delay product for an AES Sbox (component of an encryption algorithm) using different architectures



# Where do constraints come from?

- ❑ Constraints are defined by the application domain or the system that will integrate a given digital design



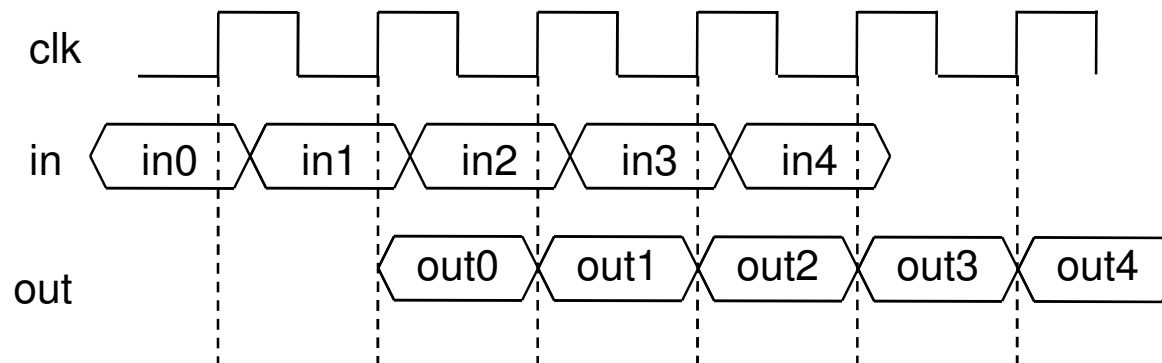
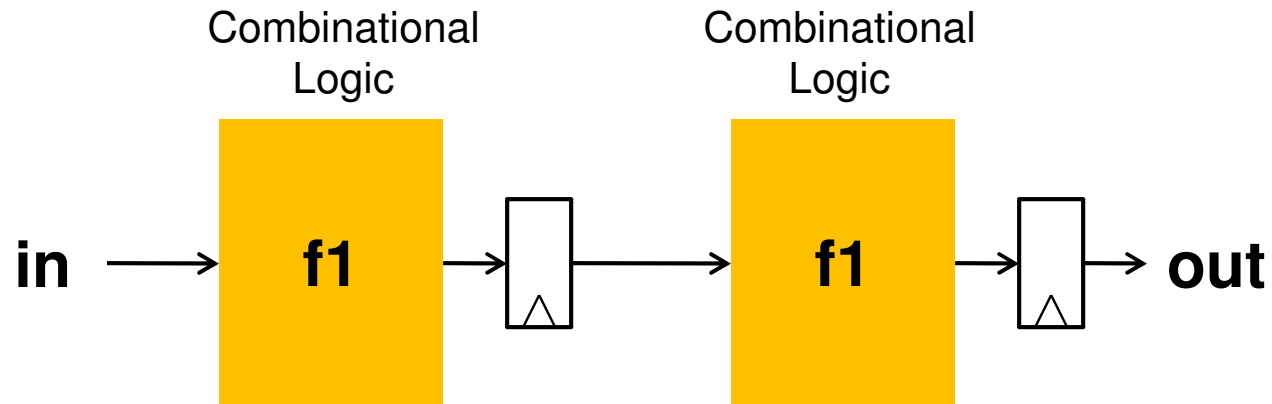
# Area Optimization - Overview

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- ❑ Resource Sharing
- ❑ Hardware Sharing Factor
- ❑ Examples:
  - An unshared multiplier
  - A shared multiplier
  - Multiplication with a constant

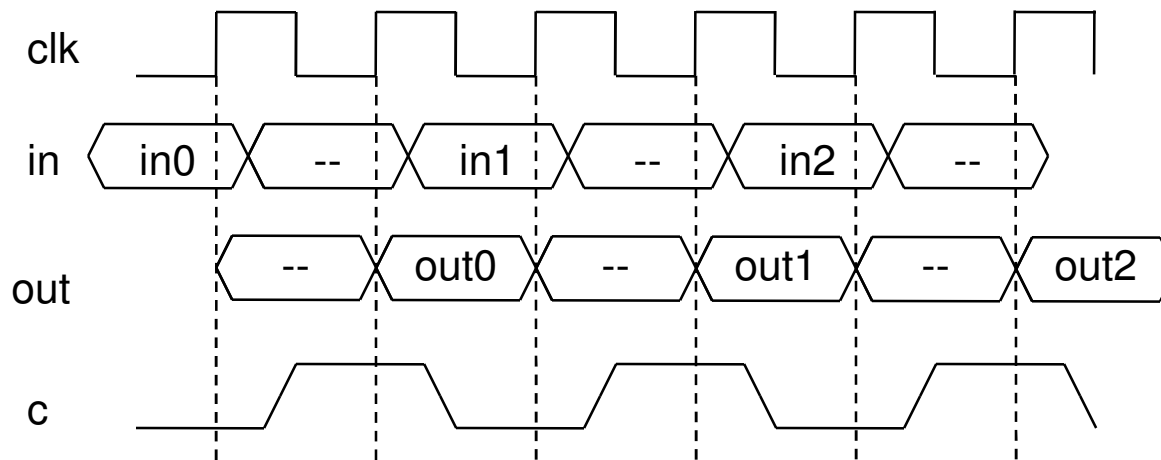
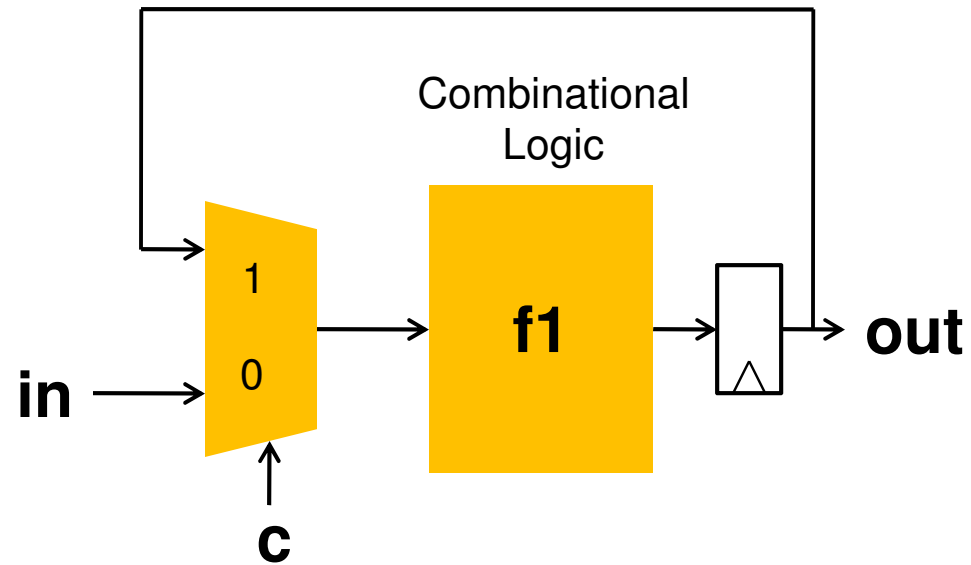
# Resource Sharing

## □ Unshared case



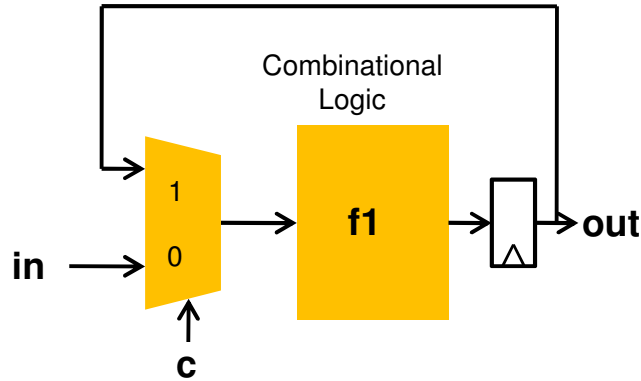
# Resource Sharing

## Shared case

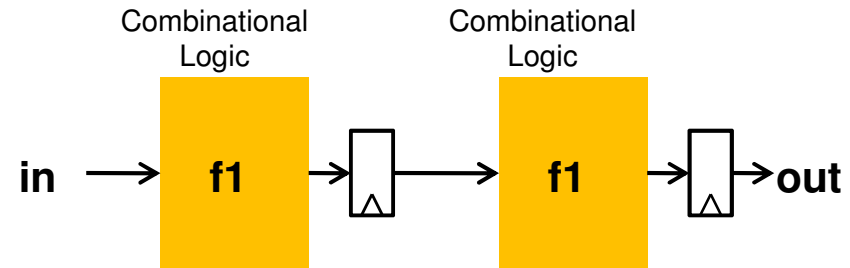




# Resource Sharing



**AREA** 1x function f1  
1 multiplexer  
1 register



**AREA** 2x function f1  
2 registers

There will be area savings in the datapath when

$$A_{f1} + A_{mux} + A_{reg} < 2A_{f1} + 2A_{reg}$$

or

$$A_{f1} + A_{reg} > A_{mux}$$

the c signal  
needs to be  
created as well

# Resource Sharing

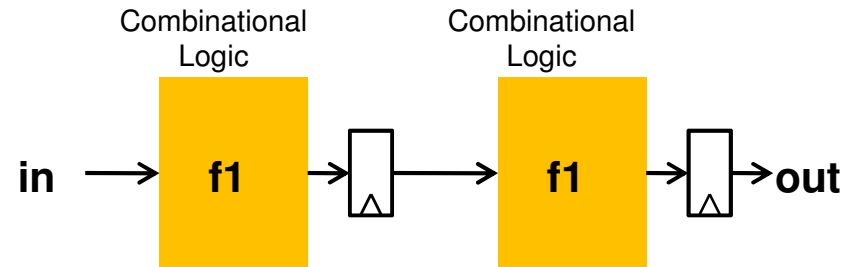
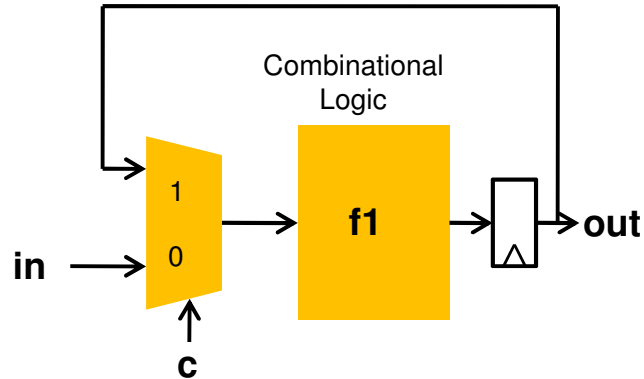
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- ❑ Registers are relatively big compared to combinational logic (recall: 6 gates for a D flip-flop); while multiplexers are small (3 gates for a mux)
- ❑ Therefore, the equation is easy to satisfy

**There will be area savings in the datapath when**

$$A_{f1} + A_{reg} > A_{mux}$$

# Resource Sharing trade-off area & performance



**AREA**

1x function f1  
1 multiplexer  
1 register

**Smaller**

2x function f1  
2 registers

**LATENCY**

Total compute time per  
input is two cycles

Total compute time per  
input is two cycles

**THROUGHPUT**

1 input each  
two cycles

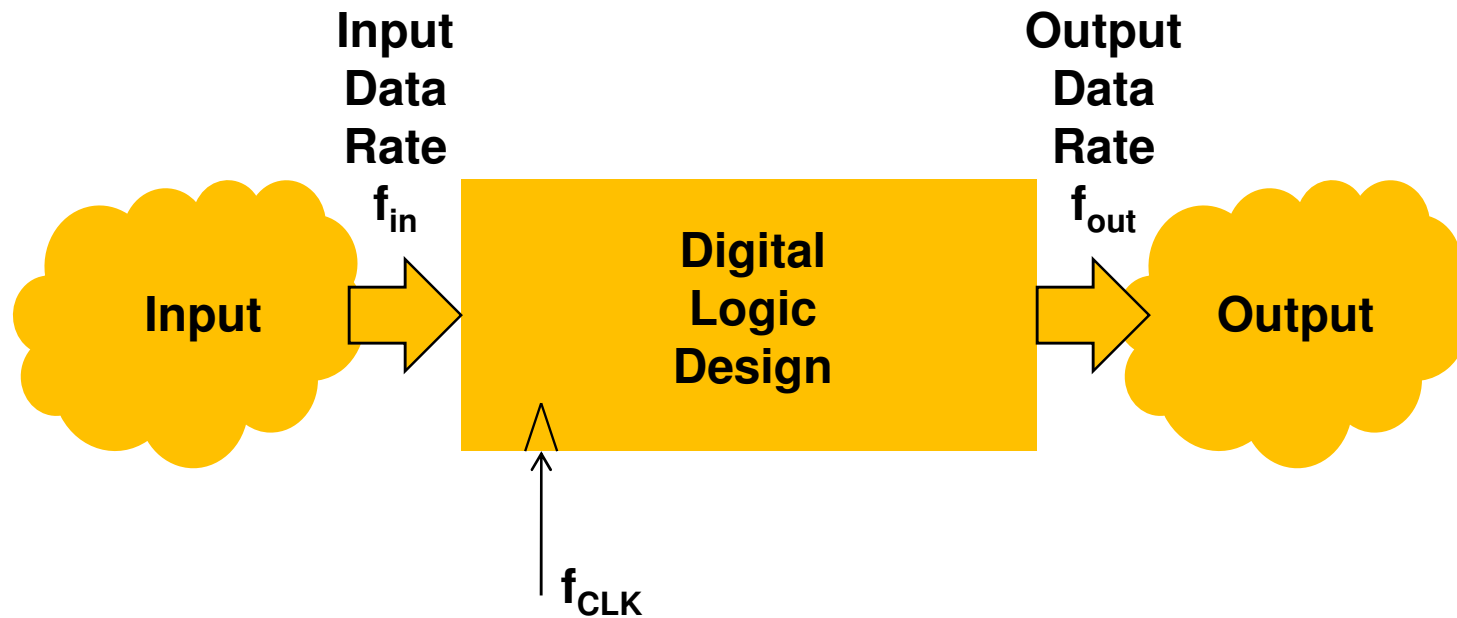
1 input each cycle

**Faster**

# Hardware Sharing Factor

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- The Hardware Sharing Factor (HSF) expresses the *potential* amount of resource sharing in a digital design

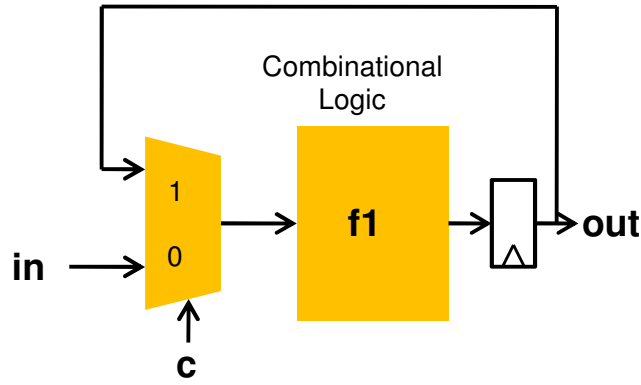


$$HSF = f_{CLK} / \max(f_{in}, f_{out})$$

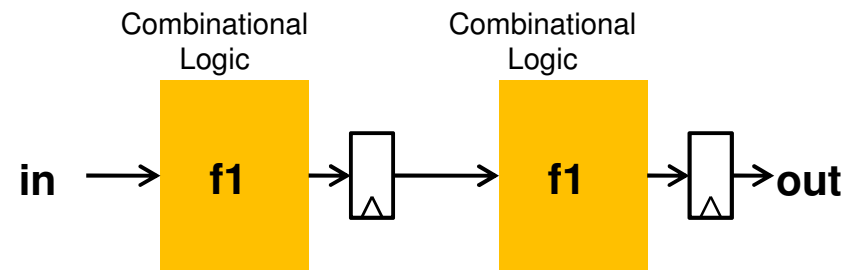
**HSF is the amount of clock cycles available per data item**

# Hardware Sharing Factor

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**HSF = 2**

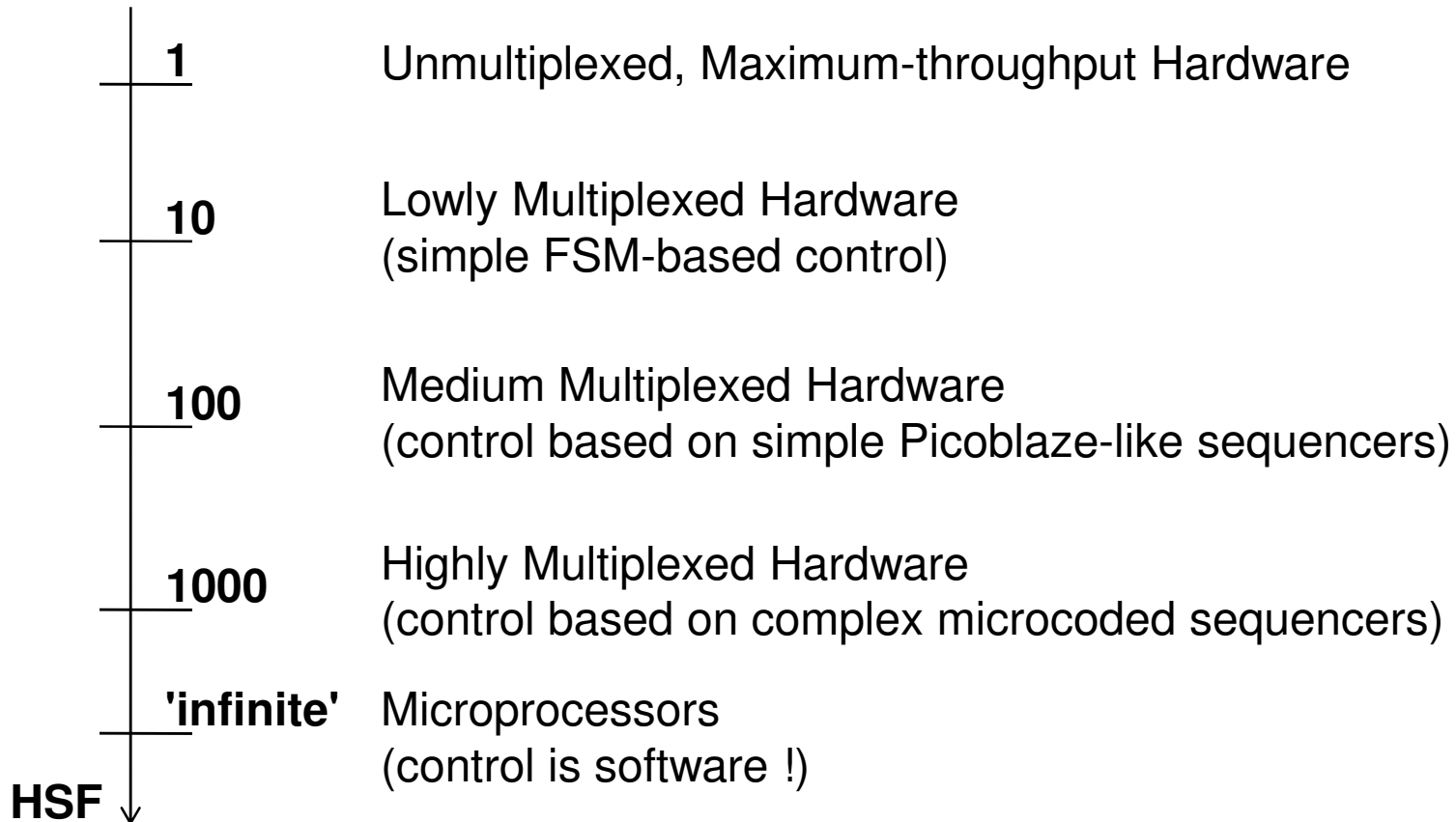


**HSF = 1**

# Hardware Sharing Factor

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- Different styles of design will be reflected with a different HSF



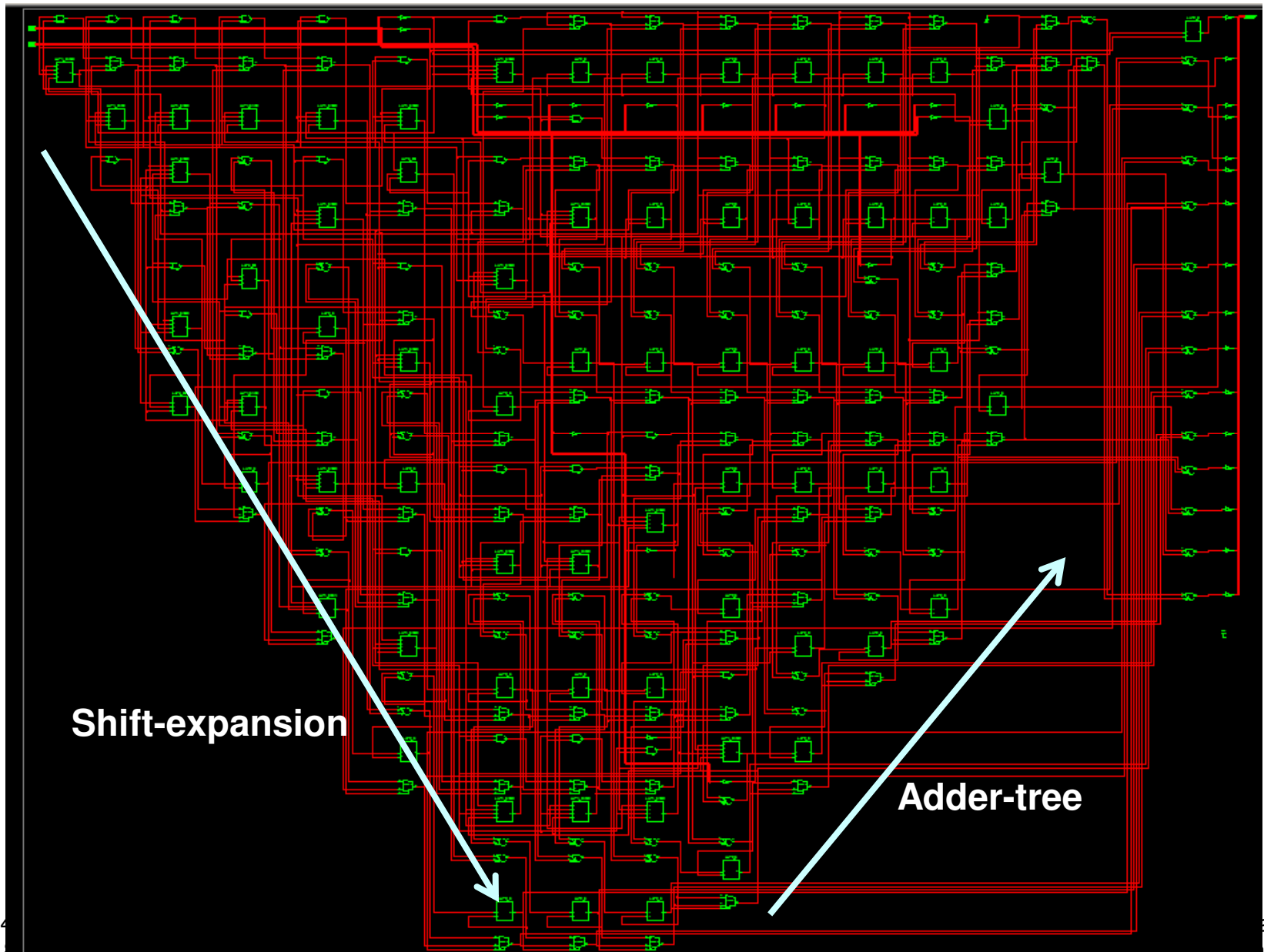
## Example: An unshared multiplier

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- Synthesize this multiplication into FPGA with multiplication 'expanded' into LUTs (as add-shift)

```
(* mult_style = "lut" *)  
  
module syn_ex(q, a, b);  
    output [15:0] q;  
    input [7:0] a, b;  
  
    assign q = a * b;  
  
endmodule
```

# Spartan3: 70 LUTs, 10.6 ns delay

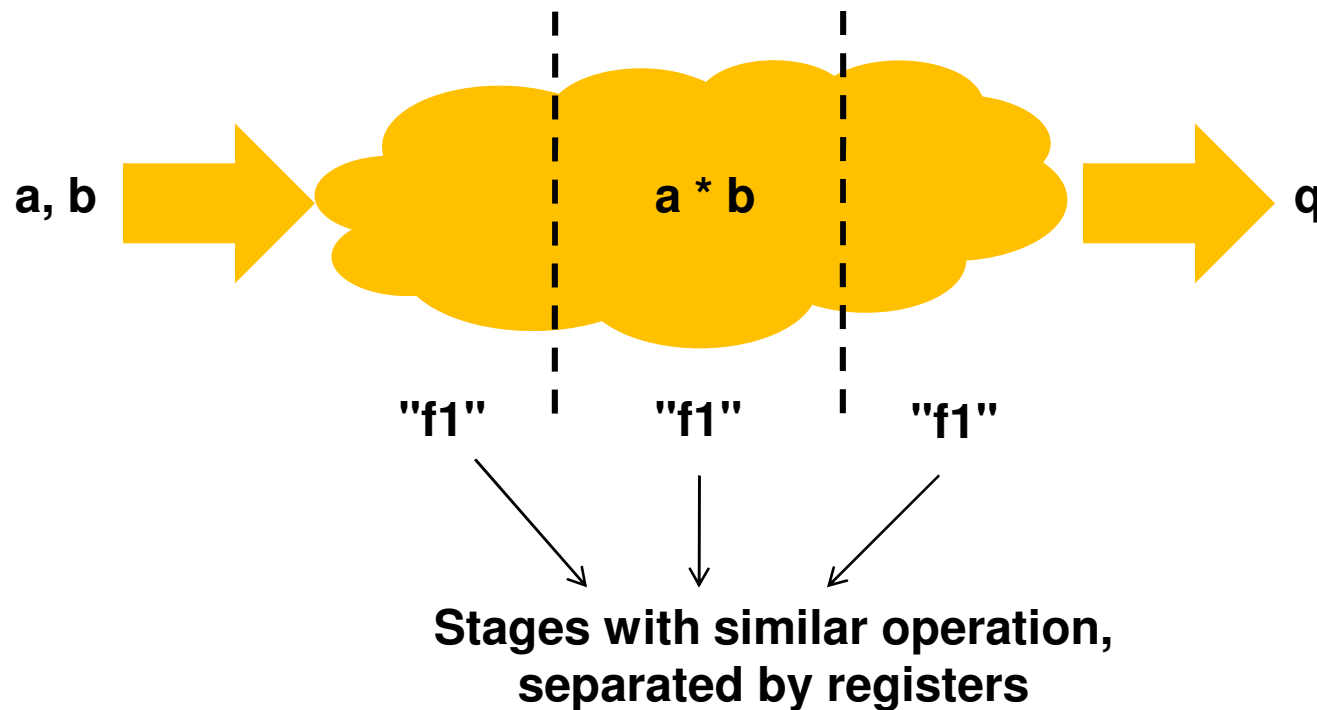




# Example: An unshared multiplier

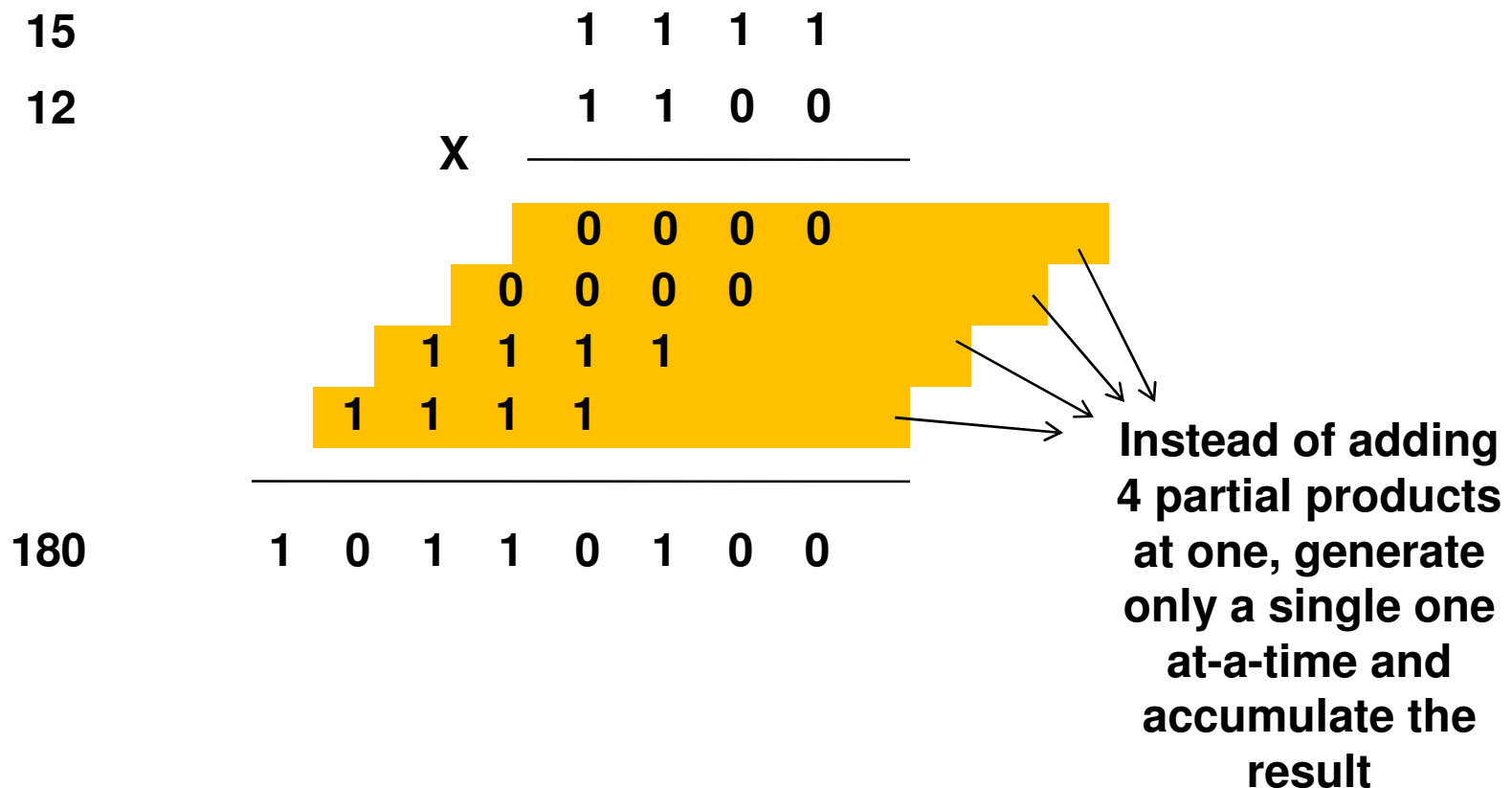
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- The idea of a shared implementation is to 'chop' the combinational logic in smaller *similar* pieces, and execute these similar pieces over multiple clock cycles



# Example: An unshared multiplier

- For a multiplication, repeated add-shift is the obvious 'repeating' part to be shared.



# Example: An unshared multiplier

---

```
module syn_ex3(q, a, b);  
    output [15:0] q;  
    reg [15:0] q;  
    input [7:0] a, b;
```

**8 \* 8 bit multiplication  
written using add-shift**

```
    reg [15:0] tmp [7:0];
```

```
    always @(a or b) begin
```

```
        tmp[0] = b[0] ? a : 15'b0;
```

```
        tmp[1] = tmp[0] + (b[1] ? {a, 1'b0} : 15'b0);
```

```
        tmp[2] = tmp[1] + (b[2] ? {a, 2'b0} : 15'b0);
```

```
        tmp[3] = tmp[2] + (b[3] ? {a, 3'b0} : 15'b0);
```

```
        tmp[4] = tmp[3] + (b[4] ? {a, 4'b0} : 15'b0);
```

```
        tmp[5] = tmp[4] + (b[5] ? {a, 5'b0} : 15'b0);
```

```
        tmp[6] = tmp[5] + (b[6] ? {a, 6'b0} : 15'b0);
```

```
        tmp[7] = tmp[6] + (b[7] ? {a, 7'b0} : 15'b0);
```

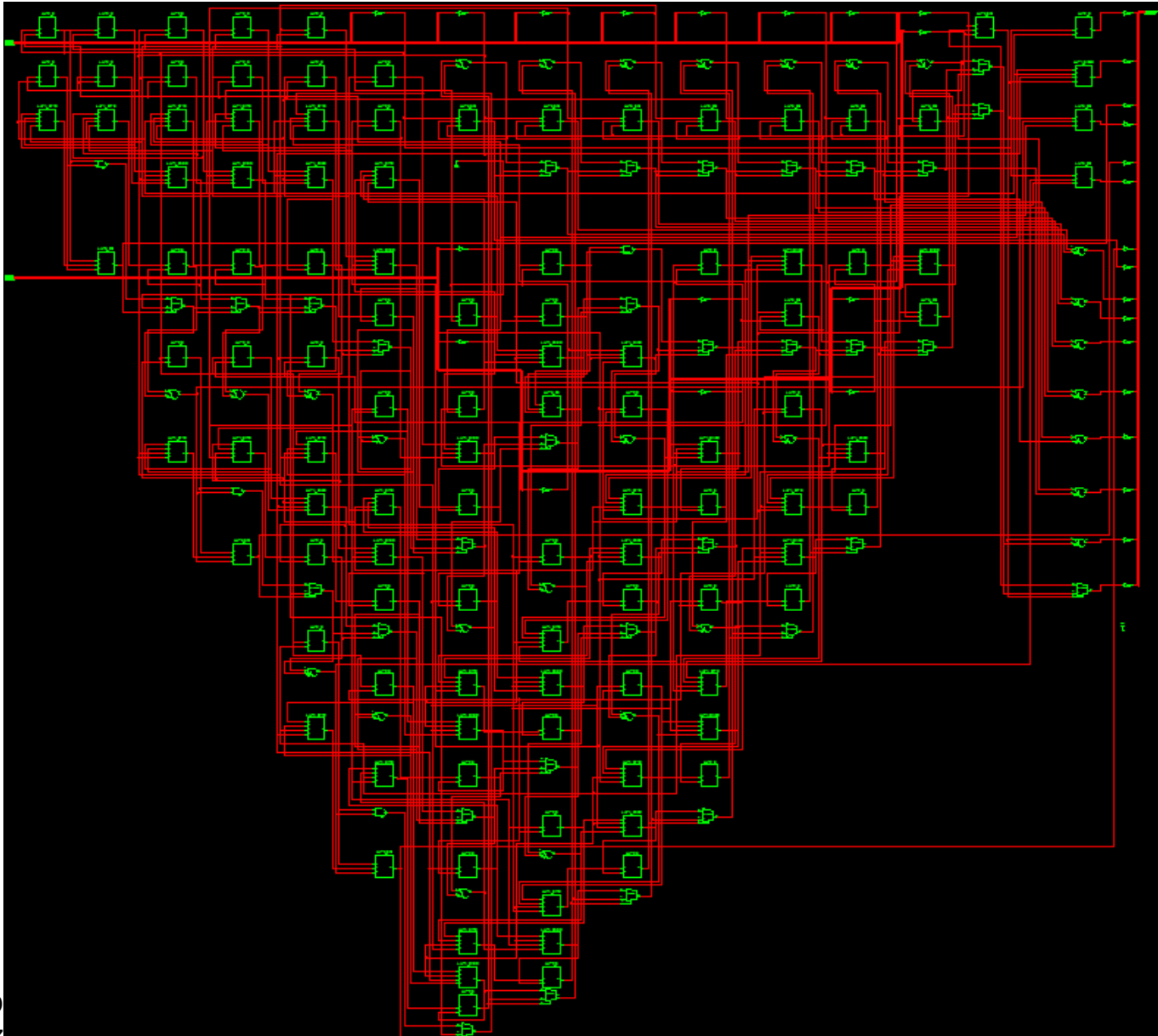
```
        q = tmp[7];
```

```
    end
```

```
endmodule
```

# Spartan3: 108 LUTs, 20ns delay ..

---



# Shared multiplier

```

module syn_ex3(q, a, b);
  output [15:0] q;
  reg [15:0] q;
  input [7:0] a, b;

```

```

reg [15:0] tmp [7:0];

```

```

always @(a or b) begin

```

```

  tmp[0] = b[0] ? a : 15'b0;

```

```

  tmp[1] = tmp[0] + (b[1] ? {a, 1'b0} : 15'b0);

```

```

  tmp[2] = tmp[1] + (b[2] ? {a, 2'b0} : 15'b0);

```

```

  tmp[3] = tmp[2] + (b[3] ? {a, 3'b0} : 15'b0);

```

```

  tmp[4] = tmp[3] + (b[4] ? {a, 4'b0} : 15'b0);

```

```

  tmp[5] = tmp[4] + (b[5] ? {a, 5'b0} : 15'b0);

```

```

  tmp[6] = tmp[5] + (b[6] ? {a, 6'b0} : 15'b0);

```

```

  tmp[7] = tmp[6] + (b[7] ? {a, 7'b0} : 15'b0);

```

```

  q = tmp[7];

```

```

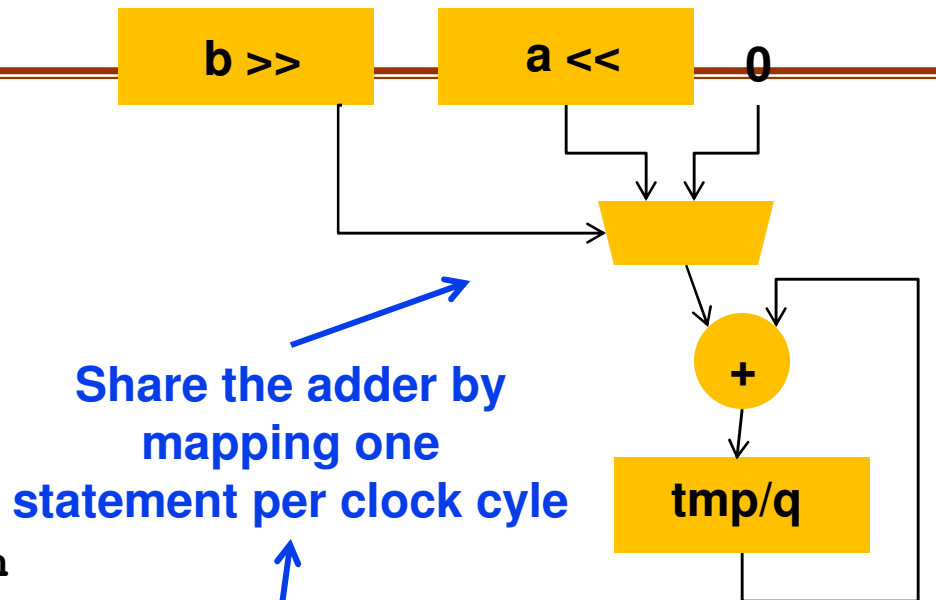
end

```

```

endmodule

```



Share the adder by  
mapping one  
statement per clock cycle

# Shared multiplier

---

```
module syn_ex4(q, done, a, b, start, clk);
    output [15:0] q;
    reg [15:0] q;
    output done;
    input [ 7:0] a, b;
    input start;
    input clk;
    reg [ 4:0] ctr;
    reg [15:0] shiftA;
    reg [ 7:0] shiftB;
    always @(posedge clk) begin
        ctr    <= (start) ? 5'b0 : ((!done) ? ctr + 1 : ctr);
        q      <= (start) ? 15'b0 :
                (!done) ? (shiftB[0] ? q + shiftA : q) : q;
        shiftB <= (start) ? b : {1'b0, shiftB[7:1]};
        shiftA <= (start) ? a : {shiftA[14:0], 1'b0};
    end
    assign done = ctr[3];
endmodule
```

# Shared multiplier

---

```
module syn_ex4(q, done, a, b, start, clk);
    output [15:0] q;
    reg [15:0] q;
    output done;
    input [7:0] a, b;
    input start;
    input clk;
    reg [ 4:0] ctr;
    reg [15:0] shiftA;
    reg [ 7:0] shiftB;
    always @(posedge clk) begin
        ctr    <= (start) ? 5'b0 : ((!done) ? ctr + 1 : ctr);
        q      <= (start) ? 15'b0 :
                (!done) ? (shiftB[0] ? q + shiftA : q) : q;
        shiftB <= (start) ? b : {1'b0, shiftB[7:1]};
        shiftA <= (start) ? a : {shiftA[14:0], 1'b0};
    end
    assign done = ctr[3];
endmodule
```

**Controller**

# Shared multiplier

---

```
module syn_ex4(q, done, a, b, start, clk);
    output [15:0] q;
    reg [15:0] q;
    output done;
    input [7:0] a, b;
    input start;
    input clk;
    reg [4:0] ctr;
    reg [15:0] shiftA;
    reg [7:0] shiftB;
    always @(posedge clk) begin
        ctr <= (start) ? 5'b0 : ((!done) ? ctr + 1 : ctr);
        q <= (start) ? 15'b0 :
            (!done) ? (shiftB[0] ? q + shiftA : q) : q;
        shiftB <= (start) ? b : {1'b0, shiftB[7:1]};
        shiftA <= (start) ? a : {shiftA[14:0], 1'b0};
    end
    assign done = ctr[3];
endmodule
```

**B shifts down, A shifts up**



# Shared multiplier

---

```
module syn_ex4(q, done, a, b, start, clk);
    output [15:0] q;
    reg [15:0] q;
    output done;
    input [7:0] a, b;
    input start;
    input clk;
    reg [4:0] ctr;
    reg [15:0] shiftA;
    reg [7:0] shiftB;
    always @(posedge clk) begin
        ctr <= (start) ? 5'b0 : ((!done) ? ctr + 1 : ctr);
        q <= (start) ? 15'b0 :
            (!done) ? (shiftB[0] ? q + shiftA : q) : q;
        shiftB <= (start) ? b : {1'b0, shiftB[7:1]};
        shiftA <= (start) ? a : {shiftA[14:0], 1'b0};
    end
    assign done = ctr[3];
endmodule
```

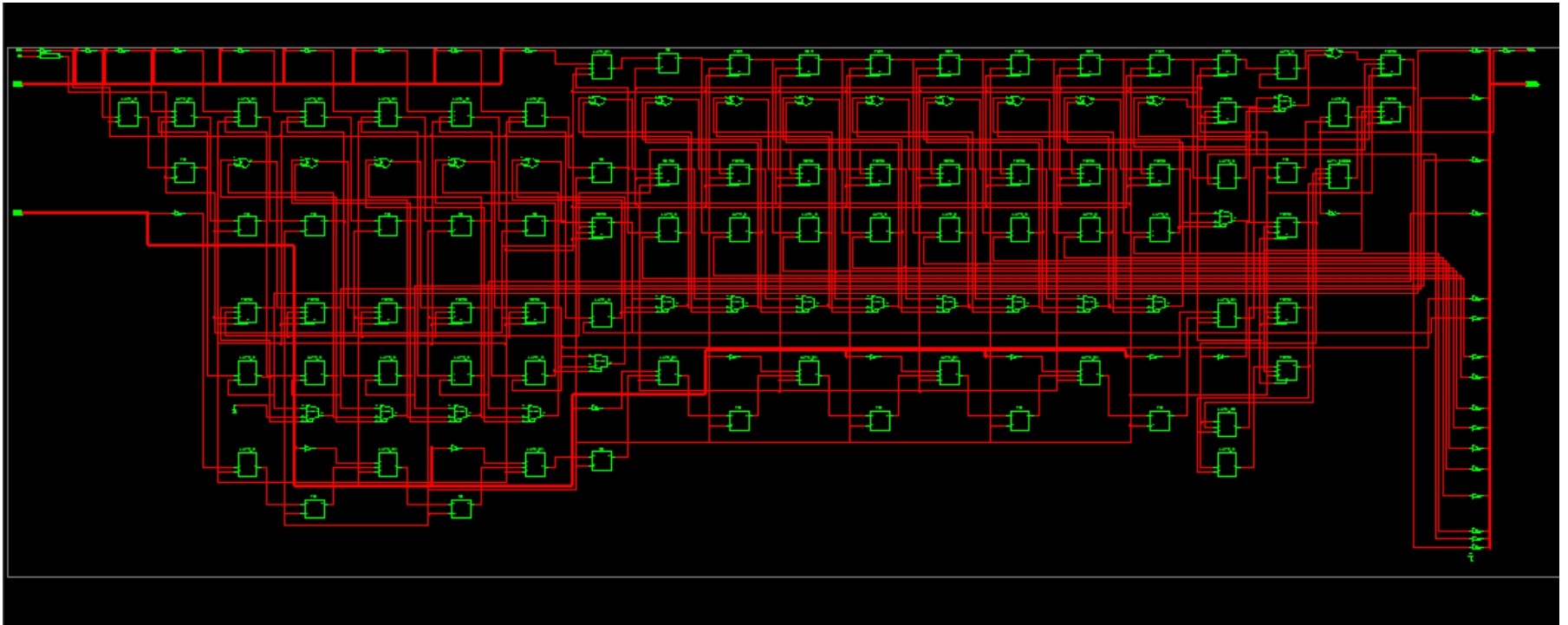
**Accumulator**

# Spartan3: 36 LUTs, 44 FF, 4.0 ns delay

---

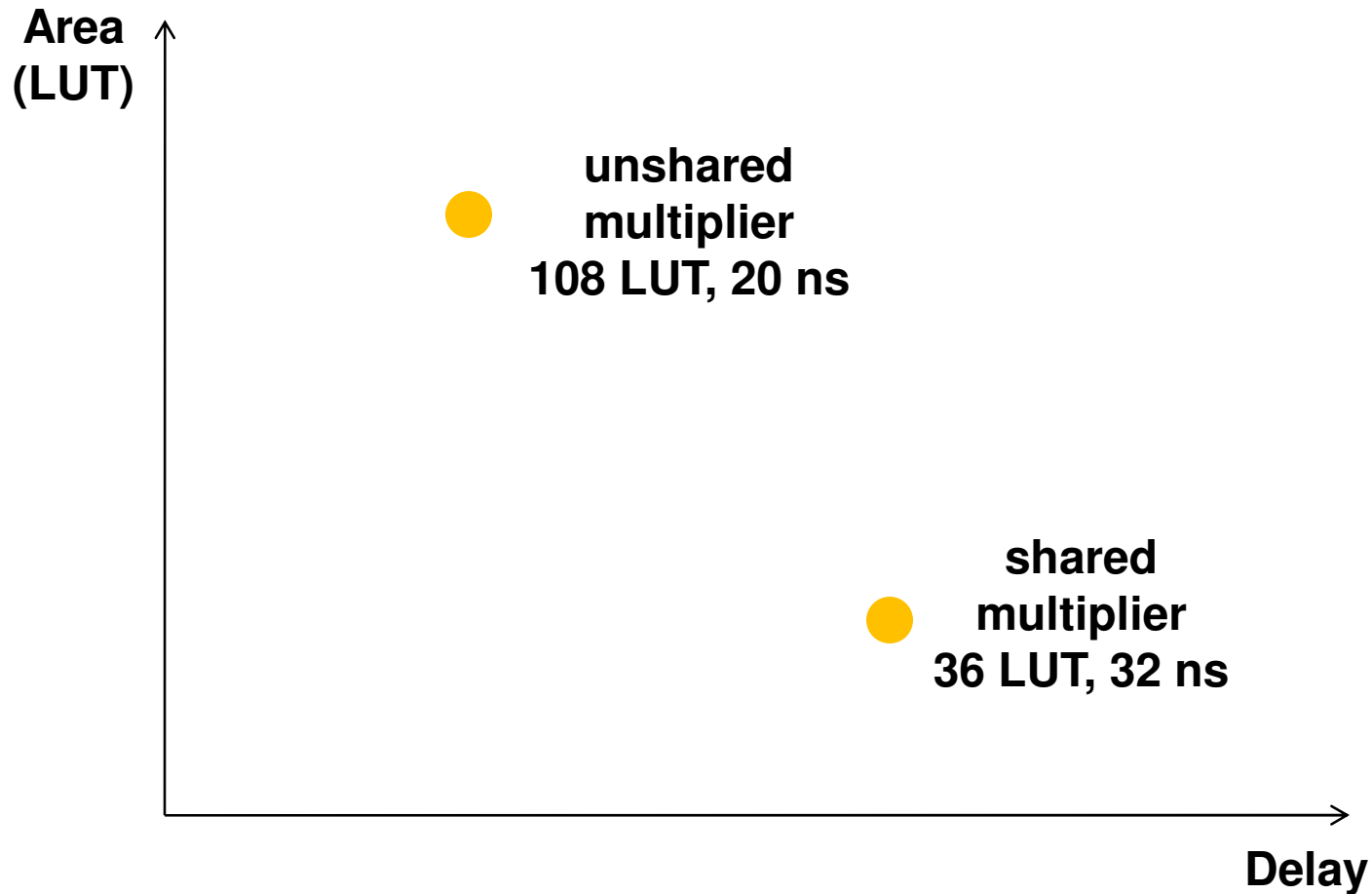


Full Multiplication still takes  $8 * 4 = 32\text{ns}$



# Sharing may reduce area at expense of delay

---



# Other Solutions are possible ...

---

```
module syn_ex3(q, a, b);  
  output [15:0] q;  
  reg [15:0] q;  
  input [7:0] a, b;
```

Create an implementation  
with two adders

```
  reg [15:0] tmp [7:0];
```

Rewrite code to two  
statements per clock cycle

```
  always @(a or b) begin
```

```
    tmp[0] = b[0] ? a : 15'b0;
```

```
    tmp[1] = tmp[0] + (b[1] ? {a, 1'b0} : 15'b0);
```

```
    tmp[2] = tmp[1] + (b[2] ? {a, 2'b0} : 15'b0);
```

```
    tmp[3] = tmp[2] + (b[3] ? {a, 3'b0} : 15'b0);
```

```
    tmp[4] = tmp[3] + (b[4] ? {a, 4'b0} : 15'b0);
```

```
    tmp[5] = tmp[4] + (b[5] ? {a, 5'b0} : 15'b0);
```

```
    tmp[6] = tmp[5] + (b[6] ? {a, 6'b0} : 15'b0);
```

```
    tmp[7] = tmp[6] + (b[7] ? {a, 7'b0} : 15'b0);
```

```
    q = tmp[7];
```

```
  end
```

```
endmodule
```

# Other Solutions are possible ...

---

```
module syn_ex3(q, a, b);  
  output [15:0] q;  
  reg [15:0] q;  
  input [7:0] a, b;
```

Create an implementation  
with four adders

```
  reg [15:0] tmp [7:0];
```

Rewrite code to four  
statements per clock cycle

```
  always @(a or b) begin
```

```
    tmp[0] = b[0] ? a : 15'b0;
```

```
    tmp[1] = tmp[0] + (b[1] ? {a, 1'b0} : 15'b0);
```

```
    tmp[2] = tmp[1] + (b[2] ? {a, 2'b0} : 15'b0);
```

```
    tmp[3] = tmp[2] + (b[3] ? {a, 3'b0} : 15'b0);
```

```
    tmp[4] = tmp[3] + (b[4] ? {a, 4'b0} : 15'b0);
```

```
    tmp[5] = tmp[4] + (b[5] ? {a, 5'b0} : 15'b0);
```

```
    tmp[6] = tmp[5] + (b[6] ? {a, 6'b0} : 15'b0);
```

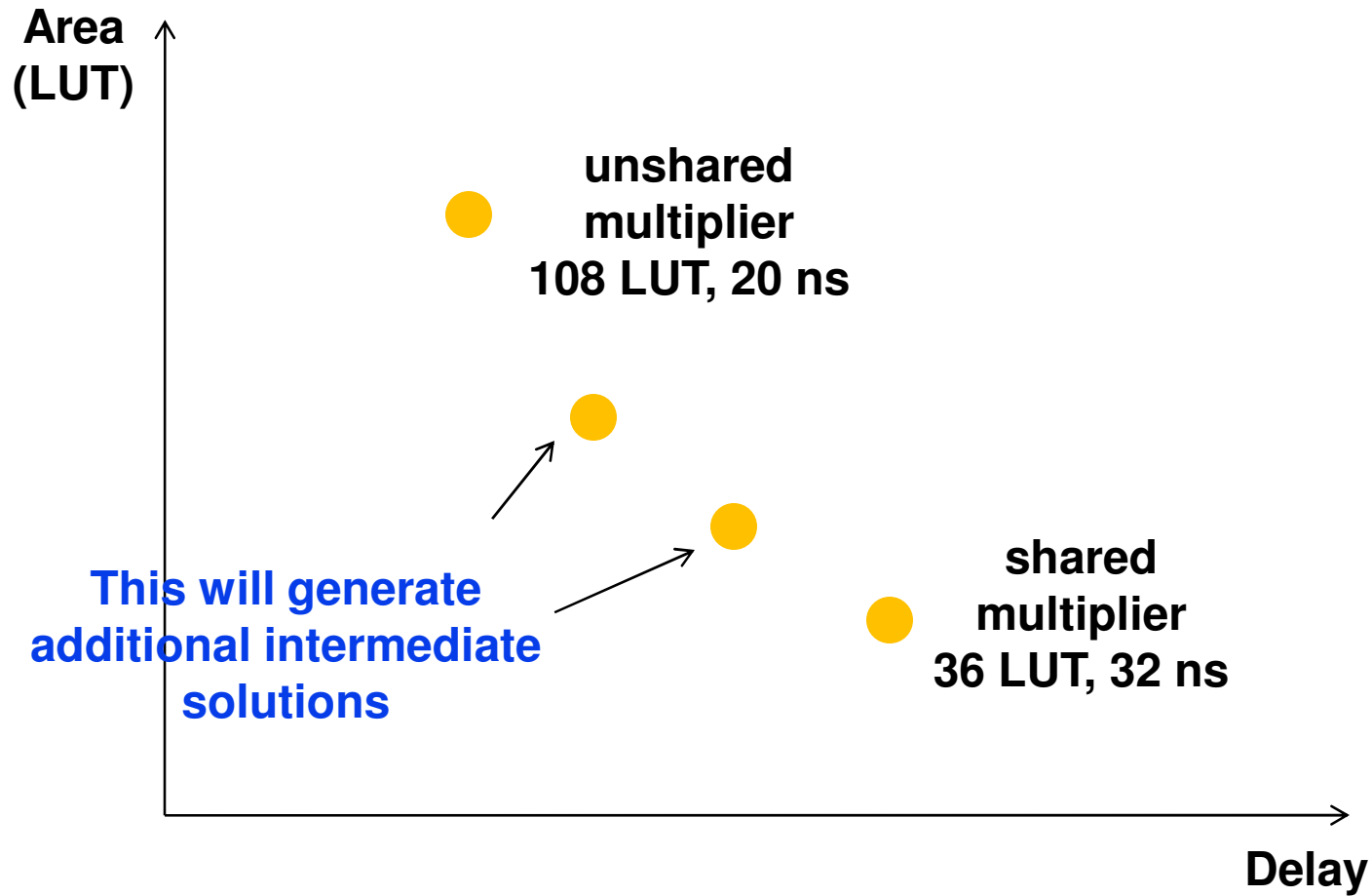
```
    tmp[7] = tmp[6] + (b[7] ? {a, 7'b0} : 15'b0);
```

```
    q = tmp[7];
```

```
  end
```

```
endmodule
```

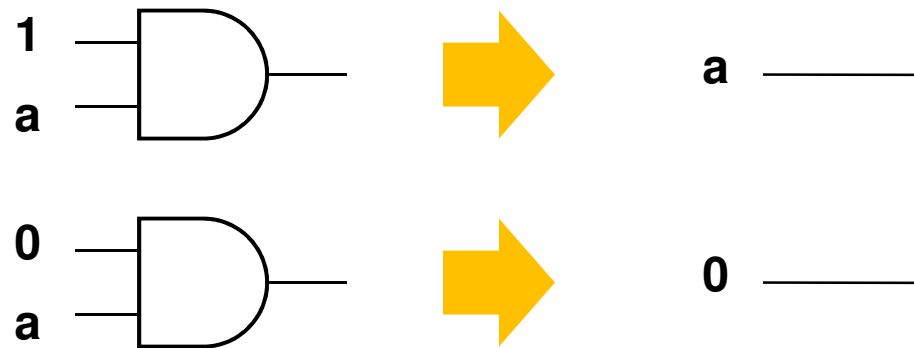
# Sharing may reduce area at expense of delay



# Multiplication with a constant

---

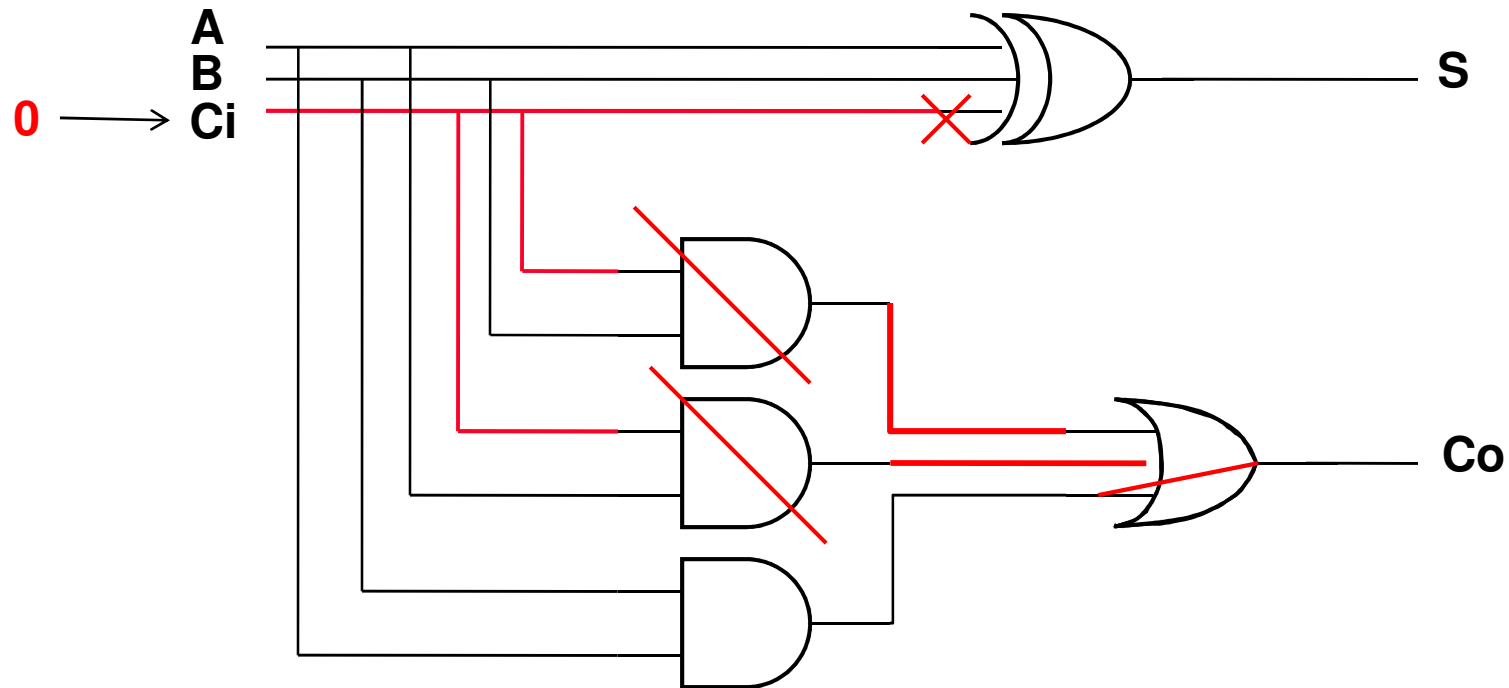
- When constants are used in a digital design, they can be propagated 'into' the implementation



- This can be used to simplify architectures considerably
- For example, a multiplier can be adapted to a more optimal design

# Example Full Adder -> Half Adder

- Propagate '0' into Ci



$$\text{Half Adder: } S = A \wedge B, \text{ Co} = A \& B$$

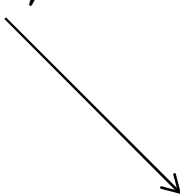


# Example: constant multiplication

---

```
(* mult_style = "lut" *)  
  
module syn_ex6(q, a);  
    output [15:0] q;  
    input [7:0] a;  
  
    assign q = a * 215;  
  
endmodule
```

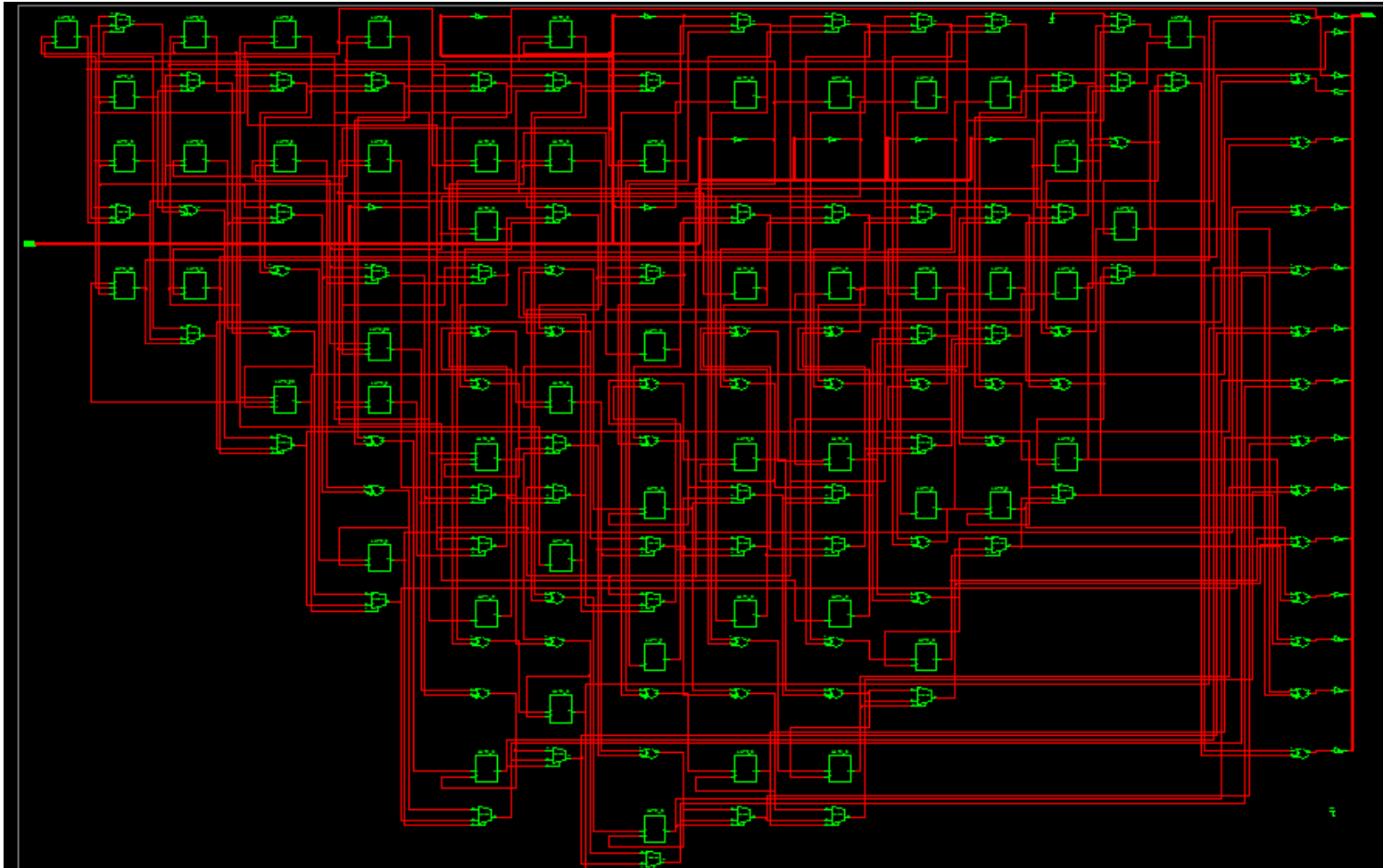
**215 = 11010111**



# Spartan3: 36 LUT, 13.5 ns

---

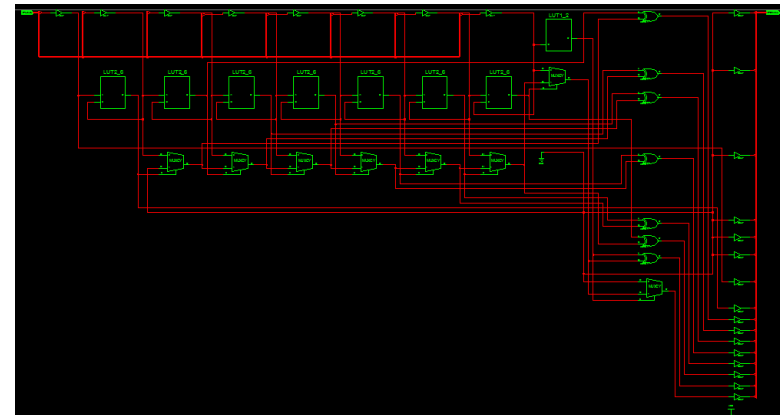
- Cfr: 70 LUT, 10ns for full 8\*8 bit



# Example: constant multiplication

```
(* mult_style = "lut" *)  
  
module syn_ex6(q, a);  
    output [15:0] q;  
    input [7:0] a;  
  
    assign q = a * 192;  
  
endmodule
```

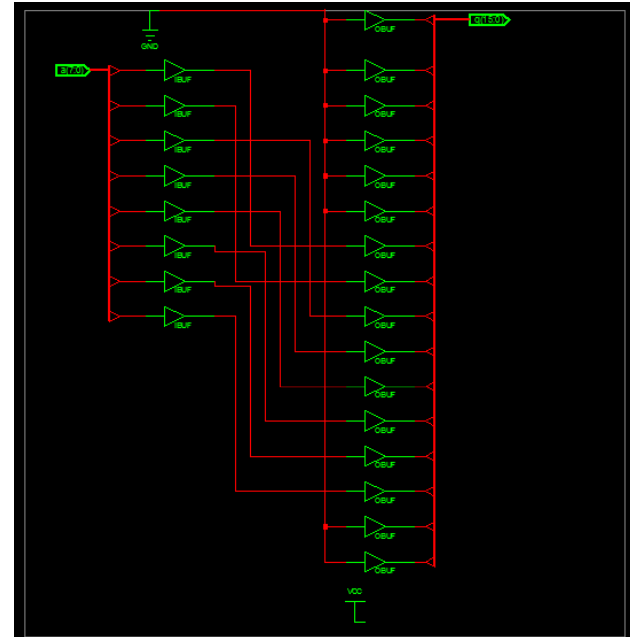
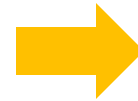
7 LUT, 10ns



**Simple constants (from a bitpattern-perspective)  
generate simple multiplier hardware ..**

# Example: constant multiplication

```
(* mult_style = "lut" *)  
  
module syn_ex6(q, a);  
    output [15:0] q;  
    input [7:0] a;  
  
    assign q = a * 64;  
  
endmodule
```



**When constant is a power of two,  
multiplication becomes a  
hardwired shift: no LUT's needed at all!**